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Computational homogenization and model order reduction of pressure diffusion in fractured rock

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Micro Abstract
Pressure diffusion in fracture networks is the dominating physical process that causes attenuation of mechanical waves traveling trough fluid-saturated rock. We simulate this process in a multi-scale approach where pressure diffusion occurs on the sub-scale and the related seismic attenuation is observed on the macro-scale. We introduce a computational homogenization scheme and develop a NTFA-type model order reduction technique which allows to derive the macroscopic properties of fractured rock.

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Introduction

Attenuation of seismic waves in fluid-saturated porous and fractured rock is of enormous interest for the exploration of geothermal and hydrocarbon reservoirs. Attenuation is caused by local pressure gradients and, therefore, local fluid transport induced by the seismic wave. This transport is dissipative, and part of the wave energy is lost. In this paper we develop computational homogenization and order reduction techniques to simulate pressure diffusion processes in fracture networks. We derive a macroscopic substitute model and investigate the seismic attenuation observed in a simple 3D network. Moreover, we show that the macroscopic model is of the generalized Maxwell-Zener type.

1 Pressure diffusion in fractured rock

We introduce a periodic Representative Volume Element (RVE) occupying the cubic volume $V_\square = l^3$. The RVE contains n fluid-filled thin fractures described as two-dimensional interfaces surrounded by a three-dimensional matrix occupying V_M. The fractures are represented by the interfaces ∂F_k, $k = 1, 2, \ldots, n$, and may cross the RVE surface $\partial V_\square$ in a periodic manner. The one-dimensional intersection of the fracture ∂F_k with the RVE surface $\partial V_\square$ is called $\partial\partial F_k$. The fractures are considered to be mechanically and hydraulically open. To compute the current fracture opening $\tau_k(\mathbf{x}, t)$ of ∂F_k, $k = 1, 2, \ldots, n$, we evaluate the displacement $\mathbf{u}$ of the matrix material surrounding the fracture. We can write $\tau_k(\mathbf{x}, t) = \tau_{0,k} + [\![\mathbf{u}]\!]_{F,k} \cdot \mathbf{n}_{F,k} \; \forall \; \mathbf{x} \in \partial F_k$, where $\tau_{0,k}(\mathbf{x}) = \tau_k(\mathbf{x})|_{t=0}$ and $\mathbf{n}_{F,k} \perp \partial F_k$. Here, we use the jump condition to connect the opposite fracture surfaces as $[\![\diamond]\!]_{F,k}(\mathbf{x}) = \diamond(\mathbf{x}^+) - \diamond(\mathbf{x}^-) \; \forall \; \mathbf{x}^\pm \in \partial V_\square^{i,\pm}$ and call the internal surface of the matrix material $\partial V_\square^i$. We consider the case of a linear-elastic and impermeable rock matrix. Hence, the fluid stored in the fractures can not migrate into the rock matrix (leak-off). Hence, we write the equilibrium condition for the rock matrix as

$$\boldsymbol{\sigma} \cdot \boldsymbol{\nabla} = \mathbf{0} \quad \forall \; \mathbf{x} \in V_M \quad \text{with} \quad \boldsymbol{\sigma} = 2\,G\,\boldsymbol{\varepsilon}^{\mathrm{dev}} + 3\,K\,\boldsymbol{\varepsilon}^{\mathrm{sph}}. \tag{1}$$

Here, G and K are the shear and the bulk modulus of the rock matrix. The linear strain tensor computes as the symmetric displacement gradient $\boldsymbol{\varepsilon} = (\mathbf{u} \otimes \boldsymbol{\nabla})^{\mathrm{sym}}$ and constitutes the

stress tensor $\boldsymbol{\sigma}$ according to Eq. $(1)_2$. The spherical and deviatoric strains are defined as $\boldsymbol{\varepsilon}^{\mathrm{sph}} = (\mathrm{tr}\,\boldsymbol{\varepsilon})\,\mathbf{I}/3$ and $\boldsymbol{\varepsilon}^{\mathrm{dev}} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{sph}}$.

The fluid pressure diffusion in ∂F_k, $k = 1, 2, \ldots, n$, is described by the continuity equation

$$\boldsymbol{\nabla} \cdot \mathbf{q}_k + \dot{\Phi}_k = 0 \quad \text{with } \dot{\Phi}_k = \frac{\dot{\tau}_k}{\tau_{k,0}} + \frac{\dot{p}_k}{K^f} - \frac{\hat{q}_k}{\tau_{k,0}} \quad \text{and } \mathbf{q}_k = -\frac{\tau_{0,k}^2}{12\,\eta^{fR}}\,\boldsymbol{\nabla} p_k. \tag{2}$$

We introduce the fluid pressure p_k in ∂F_k, the fluid velocity $\mathbf{q}_k$ along ∂F_k and the fluid mass exchange between ∂F_k and further intersecting fractures ∂F_m, $m = 1, 2, k-1, k, \ldots n$. $\dot{\Phi}_k$ describes the change in the amount of fluid stored in ∂F_k. K^f is the fluid bulk modulus. We suppose a quadratic fluid velocity profile in thickness direction of the fractures. The Poiseuille flow assumption allows us to compute the effective seepage velocity $\mathbf{q}_k$ of the fluid in ∂F_k according to Darcy's law $(2)_3$ where η^{fR} is the effective dynamic fluid viscosity.

The model coupling between the matrix material in V_M and the interfaces ∂F_k is effectuated on the one hand by the term $\dot{\tau}_k/\tau_{0,k}$ in Eq. (2). On the other hand, the fluid pressure p_k in ∂F_k acts on the internal surface ∂V_M^i of the rock matrix in terms of the Neumann-type condition $\boldsymbol{\sigma} \cdot \mathbf{n} = -p_k \cdot \mathbf{n}\ \forall\ \mathbf{x} \in V_M^i$.

2 Computational homogenization and reduced order modeling

We aim to derive a macroscopic substitute model for the heterogeneous biphasic subscale problem by computational homogenization. To this end, we impose separation of scales ($L \gg l$). Moreover, we assume that fluid pressure diffusion is a local process taking place on the subscale, only. Thus, we exclude any macroscopic transport processes. This assumption is particularly reasonable for wave induced fluid pressure diffusion in the seismic frequency range. The consequence of this locality condition is that the amount of fluid stored in the RVE remains constant during excitation. We assume the RVE to be periodic and apply periodic boundary conditions [3,4]. The consequence of the locality condition for fluid pressure diffusion is that the macroscopic substitute model is one-phasic. However, the wave induced fluid pressure diffusion and, therewith, fluid redistribution on the subscale is dissipative by nature. Hence, part of the wave energy is lost. From the macroscopic view point the overall one-phasic substitute model feels a hidden (since the related process occurs on the much smaller subscale) dissipation mechanism of viscoelastic nature [2]. We put into practice the scale transition from subscale to macro scale defining the volume averaging operator $\bar{\diamond} := \langle \diamond \rangle_M + \sum_{k=1}^n \langle \diamond \rangle_{\partial F_k} = \frac{1}{V_\square} \int_{V_M} \diamond \,\mathrm{d}v + \sum_{k=1}^n \frac{1}{V_\square} \int_{\partial F_k} \diamond\, \tau_{0,k}\,\mathrm{d}a$. Here, $\bar{\diamond}$ represents the macroscopic counterpart of an appropriate subscale quantity $\diamond$. $V_M = V_\square \setminus \{V_k, k = 1, 2, \ldots, n\}$ is the volume covered by the rock matrix. It is important to remark that the volume fraction of the fractures is small compared to the volume fraction of the matrix. Hence, we can state that $V_M \approx V_\square$ and $\langle \diamond \rangle_M \approx \langle \diamond \rangle_\square$. We impose periodic boundary conditions on the RVE, solve the RVE boundary value problem and compute the macroscopic stress response as $\bar{\boldsymbol{\sigma}} := \langle \boldsymbol{\sigma} \rangle_M - \sum_{k=1}^n \langle p_k \rangle_{\partial F_k}\ \mathbf{I}$.

We can conclude so far that the balance laws in $(1)_1$, $(2)_1$ together with the periodic boundary conditions and the stress averaging rule enable us to compute the stress response of the RVE under the excitation $\bar{\boldsymbol{\varepsilon}}(t)$. We now aim to enhance the procedure in order to derive the macroscopic viscoelastic constitutive relation explicitely. We introduce a model order reduction technique in terms of a Nonuniform Transformation Field Analysis (NTFA) [1,5] of the underlying fluid pressure diffusion problem. To this end, we apply a series expansion of the pressure field $p_k(\mathbf{x}, t)$ in ∂F_k, $k = 1, 2, \ldots, n$, and state

$$p_k(\mathbf{x}, t) \approx \sum_{a=1}^{N} \xi_a(t)\, p_a(\mathbf{x}) \qquad \forall\ \mathbf{x} \in \partial F_k. \tag{3}$$

The pressure modes p_a form a set of linearly independent pressure distributions and can be gained, for example, by a Proper Orthogonal Decomposition from appropriate training computations of

the RVE problem. For practical applications it is the aim to reduce the amount of modes to a reasonably small number N. We call the scalar parameters $\xi_a(t)$ mode activity coefficients. Indeed, these mode activity parameters can be interpreted as the viscoelastic internal variables of the macroscopic substitute model. To prove this, we decompose $\boldsymbol{\varepsilon}(\mathbf{x},\, t)$ and $\boldsymbol{\sigma}(\mathbf{x},\, t)$ in V_M accordingly and insert the decomposed fields into the weak representations of the continuity equation $(2)_1$. In [3], the procedure is presented in detail. The outcome of this operations is the macroscopic constitutive relation describing the material response of the underlying RVE problem. We obtain the macroscopic equation system and write in Voigt notation

$$\underline{\underline{\nabla}}\,\bar{\underline{\sigma}} \;=\; \underline{0}, \tag{4}$$

$$\underline{\underline{\mathcal{M}}}\,\dot{\underline{\xi}} + \underline{\underline{\mathcal{A}}}\,\underline{\xi} \;=\; \underline{\underline{\mathcal{B}}}\,\dot{\bar{\underline{\varepsilon}}}, \quad \underline{\xi}(t=0) = \underline{0}. \tag{5}$$

We define the macroscopic stress in Voigt notation as $\bar{\underline{\sigma}} = \bar{\underline{\underline{C}}}^{\text{eff}}\,\bar{\underline{\varepsilon}} + \bar{\underline{\underline{C}}}^{\xi}\,\underline{\xi}$, where $\bar{\underline{\underline{C}}}^{\text{eff}}$ is the Voigt representation of the effective stiffness tensor of the dry solid RVE with empty fractures, and $\bar{\underline{\underline{C}}}^{\xi}$ is the matrix form of $\mathbf{C}_a^{\xi} = \langle \boldsymbol{\sigma}_a \rangle_M - \sum_{k=1}^{n} \langle p_k \rangle_{\partial F_k}\,\mathbf{I}$. The stress and strain vectors are given as $\bar{\underline{\sigma}} = [\bar{\sigma}_{11}, \bar{\sigma}_{22}, \bar{\sigma}_{33}, \bar{\sigma}_{12}, \bar{\sigma}_{13}, \bar{\sigma}_{23},]^T$ and $\bar{\underline{\varepsilon}} = [\bar{\varepsilon}_{11}, \bar{\varepsilon}_{22}, \bar{\varepsilon}_{33},\, 2\,\bar{\varepsilon}_{12}, 2\,\bar{\varepsilon}_{13}, 2\,\bar{\varepsilon}_{23},]^T$. The coefficient matrices in (5) are defined as

$$\mathcal{A}_{ab} = \sum_{k=1}^{n} \left\langle \frac{\tau_{0,k}^2}{12\,\eta^{fR}}\,\boldsymbol{\nabla} p_a \cdot \boldsymbol{\nabla} p_b \right\rangle_{\partial F_k}, \qquad \mathcal{B}_{ai} = -\sum_{k=1}^{n} \left\langle \frac{p_a\, T_i^0}{\tau_{0,k}} \right\rangle_{\partial F_k},$$

$$\mathcal{M}_{ab} = \langle \boldsymbol{\sigma}_a : \boldsymbol{\varepsilon}_b \rangle_M + \sum_{k=1}^{N} \left\langle \frac{p_a\, p_b}{K^f} \right\rangle_{\partial F_k}, \tag{6}$$

$a,\, b = 1,\, 2,\, \ldots,\, N$, $i = 1,\, 2,\, \ldots,\, 6$. The parameter T_i^0 represents the fracture opening under the unit deformation mode $\bar{\varepsilon}_i$. Finally, we execute a basis shift $\{\underline{\xi}\} \rightarrow \{\underline{\chi} = \underline{\underline{\mathcal{R}}}^{-1}\,\underline{\xi}\}$ in order to diagonolize the system of evolution equations (5). Hence, we evaluate the generalized eigenvalue problem for $\underline{\underline{\mathcal{A}}}$ and $\underline{\underline{\mathcal{M}}}$, compute the matrix $\underline{\underline{\mathcal{R}}}$ of eigenvectors and the spectral matrices $\underline{\underline{\mathcal{A}}}^* = \underline{\underline{\mathcal{R}}}^T\,\underline{\underline{\mathcal{A}}}\,\underline{\underline{\mathcal{R}}}$ and $\underline{\underline{\mathcal{M}}}^* = \underline{\underline{\mathcal{R}}}^T\,\underline{\underline{\mathcal{M}}}\,\underline{\underline{\mathcal{R}}}$. This allows us to rewrite (5) in form of a system of decoupled ordinary differential equations $\dot{\underline{\chi}} + \underline{\underline{\mathcal{C}}}\,\underline{\chi} = \underline{\underline{\mathcal{D}}}\,\dot{\bar{\underline{\varepsilon}}}$, $\underline{\chi}(t=0) = \underline{0}$, with $\bar{\underline{\sigma}} = \bar{\underline{\underline{C}}}^{\text{eff}}\,\bar{\underline{\varepsilon}} + \bar{\underline{\underline{C}}}^{\chi}\,\underline{\chi}$ and $\bar{\underline{\underline{C}}}^{\chi} = \bar{\underline{\underline{C}}}^{\xi}\,\underline{\underline{\mathcal{R}}}$. Moreover, we compute $\underline{\underline{\mathcal{C}}} = (\underline{\underline{\mathcal{M}}}^*)^{-1}\,\underline{\underline{\mathcal{A}}}^*$ and $\underline{\underline{\mathcal{D}}} = (\underline{\underline{\mathcal{M}}}^*)^{-1}\,\underline{\underline{\mathcal{R}}}^T\,\underline{\underline{\mathcal{B}}}$. It is important to remark that the structure of this evolution equation is identical to that one of a generalized Maxwell-Zener model with N Maxwell chains parallel to a linear spring. Hereby, the coefficients $\mathcal{C}_{aa}$, $a = 1,\, 2,\, \ldots,\, N$, ($\underline{\underline{\mathcal{C}}}$ is a diagonal matrix) represent the characteristic frequencies of the particular Maxwell chains. The coefficients $\mathcal{D}_{ai}$, $a = 1,\, 2\, \ldots,\, N$, $i = 1,\, 2,\, \ldots,\, 6$, define the sensitivity of the internal variable χ_a for a stimulation $\bar{\varepsilon}_i$.

3 Numerical example

We use the proposed computational homogenization and model order reduction technique to investigate pressure diffusion in a simple 3D fracture network consisting of two perpendicular fractures as shown in Fig. 1. We impose the uniaxial compression $\bar{\varepsilon}_{11}(t)$ in terms of a jump function. The resulting stress relaxation data is transformed into frequency domain where we evaluate the complex elastic modulus $\bar{H}_{11}(\omega) = \mathrm{d}\bar{\sigma}_{11}(\omega)/\mathrm{d}\bar{\varepsilon}_{11}(\omega)$. The storage modulus $\mathrm{Re}(\bar{H}_{11})$ and the inverse quality factor $1/Q(\bar{H}_{11}) = -\mathrm{Im}(\bar{H}_{11})/\mathrm{Re}(\bar{H}_{11})$ are plotted in Fig. 2. Hereby, the reduced basis is enriched successively by additional modes. For $n = 6$, a very good agreement with the reference solution, which is computed with full resolution on the subscale, is reached.

Conclusions

We have introduced a computational homogenization scheme that substitutes a poroelastic model accounting for local pressure diffusion in discrete fracture networks by a viscoelastic substitute

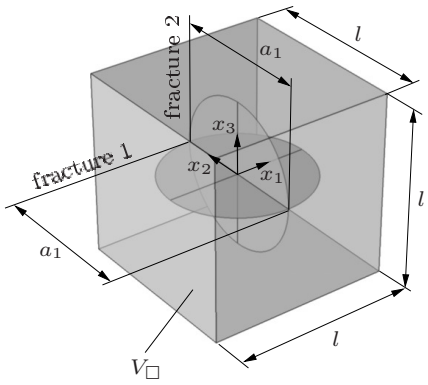

Figure 1. Cubic unit cell with two perpendicular fractures in the x_1-x_2-plane (fracture 1) and in the x_2-x_3-plane (fracture 2), $l = 10\,\text{m}$, $a_1 = 8\,\text{m}$, $\tau_0 =$1e-5 m. The material parameters are chosen as: $l = 10\,\text{n}$, $a_1 = 8\,\text{m}$, $\tau_0 =$ 1e−5 m, $G = 16\,\text{GPa}$, $K = 16\,\text{GPa}$, $K^f = 2.4\,\text{GPa}$, $\eta^{fR} = 3\,\text{mPas}$.

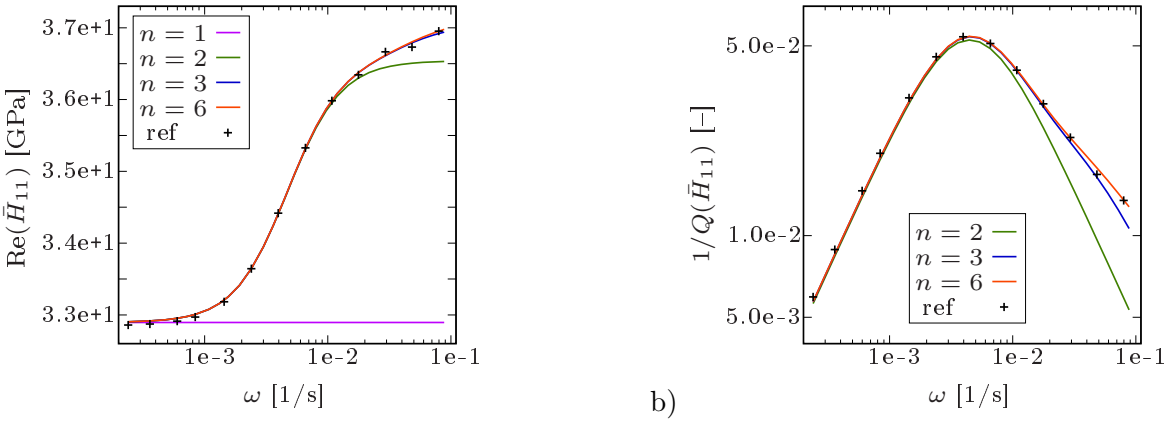

Figure 2. a) Real part and b) inverse quality factor of the P-wave coefficient $\bar{H}_{11}(\omega)$ using n internal variables.

model of the Maxwell-Zener type. The underlying model order reduction technique represents a significant enhancement of the NTFA method towards diffusion processes on interfaces. We could show that a sufficiently enriched basis of pressure modes is able to describe the macroscopic structural response with very high accuracy.

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