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An isogeometric mortar surface coupling method for trimmmed multipatch CAD geometries with application to FSI

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Micro Abstract
In this work the isogeometric analysis concept is extended to the mortar-based method for coupling of non-matching discretizations at common interfaces with application to FSI. In particular, focus is put on transferring data between real world engineering geometries modeled in CAD and low order surface discretizations. Moreover, the continuity enforcement between the trimmed multipatches is discussed.

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Introduction

Mapping methods for field transfer between non-matching interface discretizations are necessary for real-world applications with complex models and thus have been studied especially for coupled problems, such as contact problems and Fluid-Structure Interaction (FSI) simulations. In particular, a thorough study over different methods for data transfer with application to FSI is provided in [6]. The mortar-based method has shown very nice properties especially for highly non-matching discretizations. In the present contribution, the the isogeometric b-rep analysis [5] is extended to the mortar-based method in order to account for trimmed multipatch isogeometric discretizations when data should be exchanged with low order surface discretizations in the context of FSI. Additionally, a Penalty method is chosen for enforcing continuity constraints between the non-matching NURBS multipatches as well as the Dirichlet boundary conditions in a weak sense [3, 4].

1 Problem placement

Let $\Omega \subset \mathbb{R}^3$ be a multipatch surface in 3D Euclidean space, that is, Ω may be decomposed into a set of surfaces $\Omega^{(i)}$ with $i = 1, \dots, n \in \mathbb{N}$ such that,

$$\overline{\Omega} = \bigcup_{i=1}^{n} \overline{\Omega^{(i)}} \,, \tag{1a}$$

$$\Omega^{(i)} \cap \Omega^{(j)} = \emptyset \,, \text{ for all } i, j = 1, \dots n \text{ with } i \neq j \,, \tag{1b}$$

$$\Omega_{\mathrm{d}} := \bigcup_{i=1}^{n} \Omega^{(i)} \,, \tag{1c}$$

$$\overline{\Omega^{(i)}} \cap \overline{\Omega^{(j)}} = \overline{\gamma^{(i,j)}} \,, \text{ for all } i, j = 1, \dots n \text{ with } i \neq j \,, \tag{1d}$$

$$\overline{\gamma} = \bigcup_{(i,j) \in \mathcal{I}} \overline{\gamma^{(i,j)}} \,, \tag{1e}$$

$\mathcal{I}$ being the set of pairs (i, j) with $i, j = 1, \ldots, n$ excluding pairs of the form (i, i). Set of equations (1) defines a non-overlapping domain decomposition of surface Ω. Subsequently, let $\boldsymbol{\mathcal{X}} \subset \boldsymbol{\mathcal{L}}^2(\Omega)$ and $\boldsymbol{\mathcal{Y}} \subset \boldsymbol{\mathcal{L}}^2(\Omega_\mathrm{d})$ stand for all weakly continuous vector functions in Ω and in Ω_d, respectively. In other words, space $\boldsymbol{\mathcal{Y}}$ accounts also for non square integrable functions across γ. Spaces $\boldsymbol{\mathcal{X}}$ and $\boldsymbol{\mathcal{Y}}$, become inner product spaces when equipped with the inner product from the $L^2(\Omega)$ space defined as,

$$\left\langle \mathbf{d}, \widetilde{\mathbf{d}} \right\rangle_{0,\Omega} := \int_\Omega \mathbf{d} \cdot \widetilde{\mathbf{d}} \, \mathrm{d}\Omega \quad \text{for all } \mathbf{d}, \widetilde{\mathbf{d}} \in \boldsymbol{L}^2(\Omega) \ . \tag{2}$$

Clearly, every element $\mathbf{y} \in \boldsymbol{\mathcal{Y}}$ is discontinuous with discontinuous derivatives across the patch interfaces.

As a differential operator of field $\mathbf{y}$ the bending part of the rotation operator as this is defined in the Kirchhoff-Love shell with respect to the displacement field is herein chosen, see [3] for more details. In the sequel $\boldsymbol{\omega}(\mathbf{y})$ is referred to as the rotation of field $\mathbf{y}$, defined as [3],

$$\boldsymbol{\omega}(\mathbf{y}) := -\mathbf{n} \cdot \left(\boldsymbol{\nabla} y_3 + \mathbf{B} \cdot \mathbf{y}_{\|} \right) \ , \tag{3}$$

where y_3 and $\mathbf{y}_{\|}$ stand for the out of plane and the in plane components of field $\mathbf{y}$ when it is expressed over the covariant basis on surface Ω, that is,

$$\mathbf{y} = y^\alpha \mathbf{A}_\alpha + y_3 \mathbf{A}_3 = \mathbf{y}_{\|} + y_3 \mathbf{A}_3 \ , \tag{4}$$

$\mathbf{A}_\alpha$ and $\mathbf{A}_3$ being the covariant base vectors and the surface normal base vector of surface Ω. Additionally, $\mathbf{n}$, $\boldsymbol{\epsilon}$, $\boldsymbol{\nabla}$ and $\mathbf{B}$ stand for the in plane unit vector normal to the axis of rotation $\boldsymbol{\omega}$, the permutation tensor, the gradient operator and the curvature tensor in the curvilinear space of the surface Ω.

Subsequently, given two elements $\overline{\mathbf{y}} \in \boldsymbol{\mathcal{Y}}$ and $\overline{\mathbf{x}} \in \boldsymbol{\mathcal{X}}$, the task is to find another two elements $\mathbf{x} \in \boldsymbol{\mathcal{X}}$ and $\mathbf{y} \in \boldsymbol{\mathcal{Y}}$, respectively, such that,

$$\mathbf{x} = \arg \min_{\widetilde{\mathbf{x}} \in \boldsymbol{\mathcal{X}}} \|\overline{\mathbf{y}} - \widetilde{\mathbf{x}}\|_{0,\Omega} \ , \tag{5a}$$

$$\mathbf{y} = \arg \min_{\widetilde{\mathbf{y}} \in \boldsymbol{\mathcal{Y}}} \|\overline{\mathbf{x}} - \widetilde{\mathbf{y}}\|_{0,\Omega} \ , \tag{5b}$$

where $\|\bullet\|_{0,\Omega}$ stands for the norm induced by the inner product defined in (2) and where additionally the following interface and homogeneous Dirichlet boundary conditions for problems (5) are satisfied,

$$\mathbf{y}^{(i)} - \mathbf{y}^{(j)} = 0 \ , \quad \text{on each } \gamma^{(i,j)} \ , \tag{6a}$$

$$\boldsymbol{\omega}\left(\mathbf{y}^{(i)}\right) - \boldsymbol{\omega}\left(\mathbf{y}^{(j)}\right) = 0 \ , \quad \text{on each } \gamma^{(i,j)} \ , \tag{6b}$$

$$\mathbf{x} = \mathbf{x}_0 \ , \ \text{on } \Gamma_\mathrm{d} \ , \tag{6c}$$

$$\mathbf{y} = \mathbf{y}_0 \ , \ \text{on } \Gamma_\mathrm{d} \ , \tag{6d}$$

$$\boldsymbol{\omega}(\mathbf{y}) = \boldsymbol{\omega}_0 \ , \ \text{on } \Gamma_\mathrm{d} \ , \tag{6e}$$

assuming without loss of generality that all Dirichlet boundary conditions are applied along the same portion of the boundary $\Gamma_\mathrm{d} \subset \partial\Omega$. Furthermore, space $\boldsymbol{\mathcal{X}}$ is chosen such that boundary conditions (6c) are by construction satisfied, that is,

$$\boldsymbol{\mathcal{X}} := \{\mathbf{x} \in \boldsymbol{\mathcal{X}} \,|\mathbf{x} = \mathbf{x}_0 \text{ on } \Gamma_\mathrm{d}\} \ . \tag{7}$$

2 Weak formulation of the problem

To formulate the weak statement of set of problems (5) subject to conditions (6), the following normal and augmented functionals are defined,

$$L_{\mathrm{x}}(\mathbf{x}) := \frac{1}{2}\|\overline{\mathbf{y}}-\mathbf{x}\|^2_{0,\Omega} \ , \tag{8a}$$

$$L_{\mathrm{y}}(\mathbf{y}) := \frac{1}{2}\|\overline{\mathbf{x}}-\mathbf{y}\|^2_{0,\Omega} + \frac{1}{2}\left\|\overline{\alpha}_{\mathrm{y}}\boldsymbol{\chi}_{\mathrm{y}}\right\|_{0,\gamma} + \frac{1}{2}\|\overline{\alpha}_{\omega}\boldsymbol{\chi}_{\omega}\|_{0,\gamma} + \|\widetilde{\alpha}_{y}\mathbf{y}\|_{0,\Gamma_{\mathrm{d}}} + \|\widetilde{\alpha}_{\omega}\boldsymbol{\omega}\|^2_{0,\Gamma_{\mathrm{d}}} \ , \tag{8b}$$

where $\mathbf{y}^{(i)}-\mathbf{y}^{(j)} = \boldsymbol{\chi}_{\mathrm{y}}^{(i,j)} = \boldsymbol{\chi}_{\mathrm{y}}\big|_{\gamma^{(i,j)}}$ and $\omega\left(\mathbf{y}^{(i)}\right)+\omega\left(\mathbf{y}^{(j)}\right) = \boldsymbol{\omega}^{(i)}+\boldsymbol{\omega}^{(j)} = \boldsymbol{\chi}_{\omega}^{(i,j)} = \boldsymbol{\chi}_{\omega}\big|_{\gamma^{(i,j)}}$ stand for the interface jumps of fields $\mathbf{y}$ and $\boldsymbol{\omega}(\mathbf{y})$, respectively. As it can be deduced from the definition of functionals (8b) the Penalty method is employed for enforcing the interface and Dirichlet boundary conditions defined in (6a)-(6e) using correspondingly $\overline{\alpha}_{\mathrm{y}}$, $\overline{\alpha}_{\omega}$, $\widetilde{\alpha}_{\mathrm{y}}$ and $\widetilde{\alpha}_{\omega}$ as Penalty parameters for problem (5b). On the other hand, condition (6c) is enforced strongly in problem (5a) as it can be deduced by the selection of space $\boldsymbol{\mathcal{X}}$. The variational formulations of problems (8) are then obtained when the total variations δL_{x} and δL_{y} are set to zero, that is; Find $\mathbf{x}\in\boldsymbol{\mathcal{X}}$ and $\mathbf{y}\in\boldsymbol{\mathcal{Y}}$ such that,

$$a_{\mathrm{x}}(\delta\mathbf{x},\mathbf{x}) = l_{\mathrm{x}}(\delta\mathbf{x}) \ , \text{ for all } \delta\mathbf{x}\in\mathbf{X} \ , \tag{9a}$$

$$a_{\mathrm{y}}(\delta\mathbf{y},\mathbf{y}) = l_{\mathrm{y}}(\delta\mathbf{y}) \ , \text{ for all } \delta\mathbf{y}\in\mathbf{Y} \ , \tag{9b}$$

where the bilinear forms $a_{\mathrm{x}}:\boldsymbol{\mathcal{X}}\times\boldsymbol{\mathcal{X}}\to\mathbb{R}$, $a_{\mathrm{y}}:\boldsymbol{\mathcal{Y}}\times\boldsymbol{\mathcal{Y}}\to\mathbb{R}$ and the linear functionals $l_{\mathrm{x}}:\boldsymbol{\mathcal{X}}\to\mathbb{R}$, $l_{\mathrm{y}}:\boldsymbol{\mathcal{Y}}\to\mathbb{R}$ are defined as follows,

$$a_{\mathrm{x}}(\delta\mathbf{x},\mathbf{x}) := \langle\delta\mathbf{x},\mathbf{x}\rangle_{0,\Omega} \ , \tag{10a}$$

$$\begin{aligned} a_{\mathrm{y}}(\delta\mathbf{y},\mathbf{y}) :=\ & \langle\delta\mathbf{y},\mathbf{y}\rangle_{0,\Omega} + \left\langle\delta\boldsymbol{\chi}_{\mathrm{y}},\overline{\alpha}_{\mathrm{y}}\boldsymbol{\chi}_{\mathrm{y}}\right\rangle_{0,\gamma} + \langle\delta\boldsymbol{\chi}_{\omega},\overline{\alpha}_{\omega}\boldsymbol{\chi}_{\omega}\rangle_{0,\gamma} + \langle\delta\mathbf{y},\widetilde{\alpha}_{\mathrm{y}}\mathbf{y}\rangle_{0,\Gamma_{\mathrm{d}}} \\ & + \langle\delta\boldsymbol{\omega},\widetilde{\alpha}_{\omega}\boldsymbol{\omega}\rangle_{0,\Gamma_{\mathrm{d}}} \ , \end{aligned} \tag{10b}$$

$$l_{\mathrm{x}}(\delta\mathbf{x}) := \langle\delta\mathbf{x},\overline{\mathbf{y}}\rangle_{0,\Omega} \ . \tag{10c}$$

$$l_{\mathrm{y}}(\delta\mathbf{y}) := \langle\delta\mathbf{y},\overline{\mathbf{x}}\rangle_{0,\Omega} \ , \tag{10d}$$

Each of variational problems (9) have a unique solution since forms (10a), (10b) are bilinear, coercive and continuous and functionals (10c), (10d) are bounded.

3 Discretization

Let $\boldsymbol{\mathcal{X}}_{\mathrm{h}}\subset\boldsymbol{\mathcal{X}}$ and $\boldsymbol{\mathcal{Y}}_{\mathrm{h}}\subset\boldsymbol{\mathcal{Y}}$ be a Finite Element and an isogeometric discretization of spaces $\boldsymbol{\mathcal{X}}$ and $\boldsymbol{\mathcal{Y}}$, respectively. Let also $\left(\boldsymbol{\psi}_j\right)_{j=1}^{m}$ and $\left(\boldsymbol{\phi}_j^{(i)}\right)_{j=1}^{m^{(i)}}$ with $i=1,\dots,n$ be bases of spaces $\boldsymbol{\mathcal{X}}_{\mathrm{h}}$ and $\boldsymbol{\mathcal{Y}}_{\mathrm{h}}$, respectively. Subsequently, there exist constants $(\hat{x}_j)_{j=1}^{(m)}$ and $\left(\hat{y}_j^{(i)}\right)_{j=1}^{m^{(i)}}$ for each patch $i=1,\dots,n$ such that,

$$\mathbf{x} := \sum_{j=1}^{m}\boldsymbol{\psi}_j\hat{x}_j = \boldsymbol{\Psi}\hat{\mathbf{x}} \ , \tag{11a}$$

$$\mathbf{y} := \sum_{i=1}^{n}\sum_{j=1}^{m^{(j)}}\boldsymbol{\phi}_j^{(i)}\hat{y}_j^{(i)} = \begin{bmatrix}\boldsymbol{\Phi}^{(1)} & \cdots & \boldsymbol{\Phi}^{(n)}\end{bmatrix}\begin{bmatrix}\hat{\mathbf{y}^{(1)}}\\ \vdots \\ \hat{\mathbf{y}^{(n)}}\end{bmatrix} = \boldsymbol{\Phi}\hat{\mathbf{y}} \ , \tag{11b}$$

$\boldsymbol{\Psi}$, $\boldsymbol{\Phi}^{(i)}$ being the basis function matrices for the Finite Element and the isogeometric discretization for each patch, respectively. Moreover, $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}^{(i)}$ stand for the vector of Degrees of Freedom (DOFs) for the Finite Element and the isogeometric discretizations of each patch, respectively.

Then, $\mathbf{\Phi}$ and $\hat{\mathbf{y}}$ denote the basis function matrix and the vector of DOFs for the whole multipatch geometry. Subsequently, the discrete form of variational problems (9a) and (9b), respectively write,

$$\mathbf{C}_{\mathrm{xx}}\hat{\mathbf{x}} = \mathbf{C}_{\mathrm{xy}}\hat{\mathbf{y}} \ , \tag{12a}$$

$$\mathbf{C}_{\mathrm{yy}}\hat{\mathbf{y}} = \mathbf{C}_{\mathrm{yx}}\hat{\mathbf{x}} \ , \tag{12b}$$

where the involved matrices are defined as,

$$C_{\mathrm{xx},i,j} := \left\langle \boldsymbol{\psi}_i, \boldsymbol{\psi}_j \right\rangle_{0,\Omega} \ , \tag{13a}$$

$$\mathbf{C}_{\mathrm{yy}} := \begin{bmatrix} \mathbf{C}_{\mathrm{yy}}^{(1)} + \mathbf{C}_{\mathrm{p}}^{(1)} & \cdots & \mathbf{C}_{\mathrm{p}}^{(1,n)} \\ \vdots & \ddots & \vdots \\ \mathbf{C}_{\mathrm{p}}^{(1,n)} & \cdots & \mathbf{C}_{\mathrm{yy}}^{(n)} + \mathbf{C}_{\mathrm{p}}^{(n)} \end{bmatrix} , \tag{13b}$$

$$C_{\mathrm{yy},j,k}^{(i)} := \left\langle \boldsymbol{\phi}_k^{(i)}, \boldsymbol{\phi}_l^{(i)} \right\rangle_{0,\Omega} + \left\langle \boldsymbol{\phi}_k^{(i)}, \widetilde{\alpha}_{\mathrm{y}} \boldsymbol{\phi}_l^{(i)} \right\rangle_{0,\Gamma_{\mathrm{d}}} + \left\langle \boldsymbol{\omega}\left(\boldsymbol{\phi}_k^{(i)}\right), \widetilde{\alpha}_{\omega} \boldsymbol{\omega}\left(\boldsymbol{\phi}_l^{(i)}\right) \right\rangle_{0,\Gamma_{\mathrm{d}}} , \tag{13c}$$

$$C_{\mathrm{p},j,k}^{(i)} := \left\langle \boldsymbol{\phi}_j^{(i)}, \overline{\alpha}_{\mathrm{y}} \boldsymbol{\phi}_k^{(i)} \right\rangle_{0,\gamma} + \left\langle \boldsymbol{\omega}\left(\boldsymbol{\phi}_j^{(i)}\right), \overline{\alpha}_{\omega} \boldsymbol{\omega}\left(\boldsymbol{\phi}_k^{(i)}\right) \right\rangle_{0,\gamma} , \tag{13d}$$

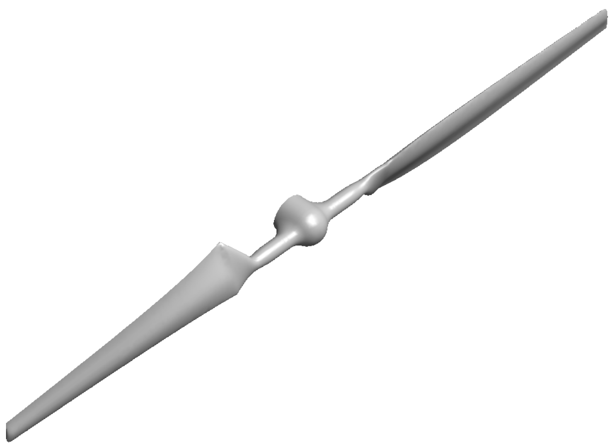
$$C_{\mathrm{p},k,l}^{(i,j)} := -\left\langle \boldsymbol{\phi}_k^{(i)}, \overline{\alpha}_{\mathrm{y}} \boldsymbol{\phi}_l^{(j)} \right\rangle_{0,\gamma} + \left\langle \boldsymbol{\omega}\left(\boldsymbol{\phi}_k^{(i)}\right), \overline{\alpha}_{\omega} \boldsymbol{\omega}\left(\boldsymbol{\phi}_l^{(j)}\right) \right\rangle_{0,\gamma} , \tag{13e}$$

$$C_{\mathrm{yx},k,l} := \sum_{i=1}^{n} \left\langle \boldsymbol{\phi}_k^{(i)}, \boldsymbol{\psi}_l \right\rangle_{0,\Omega} = C_{\mathrm{xy},l,k} \ . \tag{13f}$$

4 Numerical Results

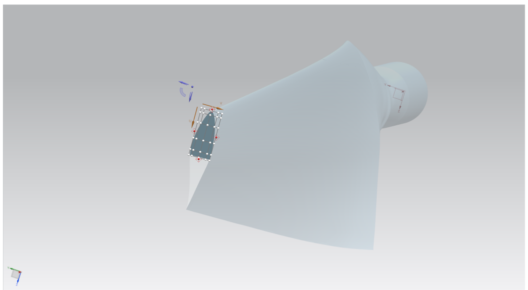
The applicability of the method is demonstrated using the blades taken from the NREL wind turbine model [1] within a fluid-structure interaction environment. The real model of the wind turbine and the corresponding CAD model of its two blades can be seen in Fig. 1. The two blade CAD model excluding the motor hub, see Fig. 1b, consists out of 64 trimmed patches. An example of two trimmed patches over the multipatch surface are given in Fig. 2. As structural

(a) Picture of the NREL wind turbine [1].

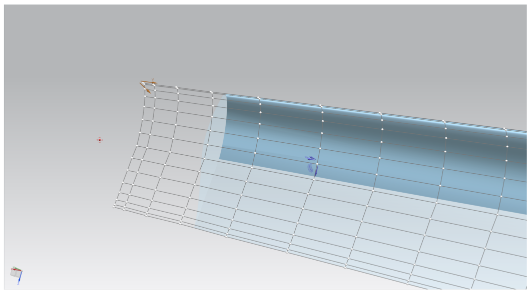


(b) CAD model of the turbine blades with the rotor hub.

Figure 1. NREL wind turbine real and CAD model.

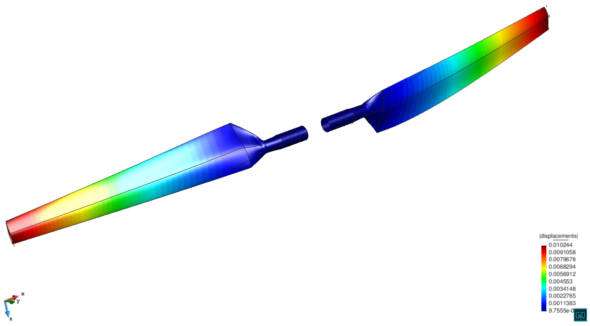


(a) Trimmed patch on the tip of one blade.

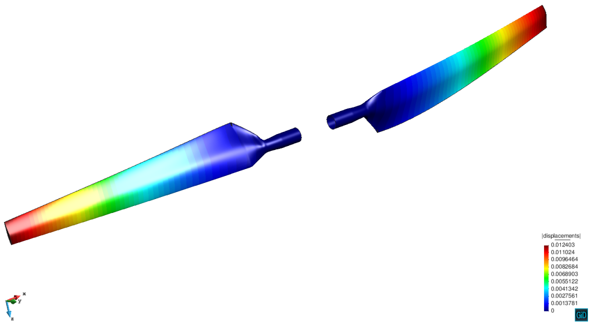


(b) Trimmed patch along one blade's surface.

Figure 2. NREL wind turbine real and CAD model.



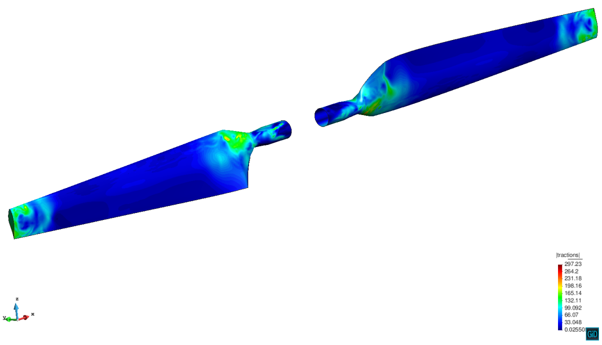
(a) Displacement field on the multipatch isogeometric structure at the 67th time step.



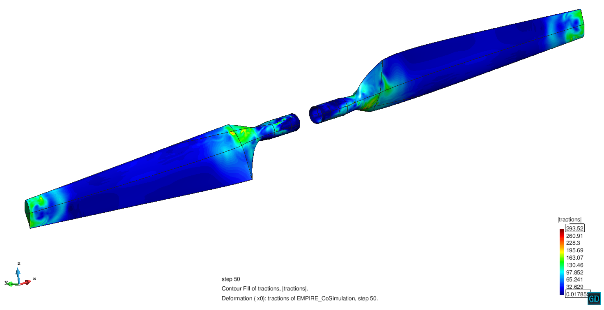
(b) Mapped displacement field on the fluid interface mesh at the 67th time step.

Figure 3. Displacement mapping solving problem (9a).

model the Kirchhoff-Love shell model is chosen, whereas the continuity constraints among the multipatches and the weak application of the Dirichlet conditions is done using the Penalty method, see [5]. The fluid is solved using the Finite Volume Method within the open-source program OpenFOAM [2]. The coupled problem is solved using the partitioned Gauss-Seidel approach. The displacements are mapped onto the fluid FSI interface, see Fig. 3b from the isogeometric multipatch structure, see Fig. 3a, solving variational problem (9a) whereas the



(a) Traction field on the fluid-interface at the 50th time step.



(b) Mapped Traction field on the multipatch structure at the 50th time step.

Figure 4. Traction mapping solving problem (9b).

traction field on the multipatch isogeometric surface, see Fig. 4b, is found when mapping the traction field from the fluid FSI interface, see Fig. 4a, solving variational problem (9b). The Penalty parameters for the enforcement of the continuity across the multipatches and the weak Dirichlet boundary conditions in the mapping equation (9b) are chosen with respect to the element size across each iterface and each Dirichlet boundary.

Conclusions

Herein an isogeometric mortar-based mapping method is proposed for its application to Fluid-Structure interaction. The method is enhanced with Penalty terms accounting for the coupling between the different patches and the weak application of Dirichlet boundary conditions. The applicability of the method is demonstrated using real world engineering problems which involve trimmed multipatch geometries thus extending the concept of the isogeometric B-rep analysis to the mortar-based method for data transfer between isogeometric and classical low order surface discretizations.

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