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Design of a Nonlinear Observer for a Very Flexible Parallel Robot

Fatemeh Ansarieshlaghi[1]* and Peter Eberhard[1]

Micro Abstract
A flexible robot in lambda configuration has been modeled and built in hardware. Since there is no direct feedback of the end-effector, a nonlinear observer to estimate the position of the end-effector is designed and implemented. The nonlinear observer results show that the end-effector position can be estimated with high accuracy. Also, using results from the nonlinear observer, the model of the robot is improved so that the maximum end-effector absolute tracking error is drastically decreased.

[1]Institute of Engineering and Computational Mechanics, University of Stuttgart, Stuttgart, Germany
***Corresponding author**: fatemeh.ansari@itm.uni-stuttgart.de

Introduction

This research investigates the design of a nonlinear observer to estimate the states of a very flexible multibody dynamic system in order to use them for the trajectory tracking and control. The observers are widely used in order to estimate the full states of a system, e.g, state estimation of the linear systems [11], the velocity estimation of the rigid joints [9], and the state estimation of the minimum phase systems [7]. Hence, a lot of simulation research on the state estimation of a single flexible link manipulator and the flexible multibody system are investigated, e.g, design of a linear observer for a flexible multibody system without the passive joint [13], the nonlinear observer design for a flexible beam [3] and for a flexible multibody system [14]. So far experimental research is focused only on the observation of a flexible beam [5] to estimate the vibration of the beam using a laser displacement sensor.

The novelty of this work is, that a high gain observer for trajectory tracking with high speed of a very flexible parallel robot is designed and the estimates converge to measurement states. In the proposed nonlinear observer, only the position and the velocity measurement of the prismatic joints and the deformation of the long flexible link are required to estimate the elastic and passive states. This nonlinear observer is simulated and implemented on the lambda robot. Based on the observer results, the model of the very flexible multibody system is improved and a new observer is redesigned. The new nonlinear observer estimates the system states and the end-effector position with high accuracy. The experimental results validation verify the accuracy of the estimation of the end-effector positions and the elastic deformation.

1 Experimental setup

A flexible robot in lambda configuration which has been modeled and built by the Institute of Engineering and Computational Mechanics of the University of Stuttgart can be seen in Figure 1a. This robot has highly flexible links as the long and short links. The end of the short link is connected to the middle of the long link with the rigid bodies. The robot has two prismatic actuators connecting the links to the ground and the control outputs apply to these two linear actuators. Also, every link is connected to a passive revolute joint that is located on the linear actuator. There is another revolute joint that is used to connect two links at the end of the short link and the middle of the long link. An additional rigid body is attached to the free end of the long link as an end-effector.

1.1 Flexible Multibody Modeling

To model the lambda robot, the modelling process can be separated into three major steps. At first, the flexible components of the system are modeled with the finite element method in the commercial finite element code *ANSYS*. Next, to have a possible on-line control, the degrees of freedom of flexible links should be decreased. Therefore, model order reduction is utilized in order to reduce the long flexible link's model [1] in the *MatMorembs* [4]. Then, all the bodies such as the rigid and flexible parts are modeled as a multibody dynamic system with a kinematic loop in the academic multibody code *Neweul-M*^{2} [8]. The equation of motion is derived by applying the Newton-Euler equation with D'Alembert's principle. For the nonlinear systems with the kinematic loop constraint, the nonlinear equation of motion with the generalized coordinates $\boldsymbol{q} \in \mathbb{R}^n$ is written as

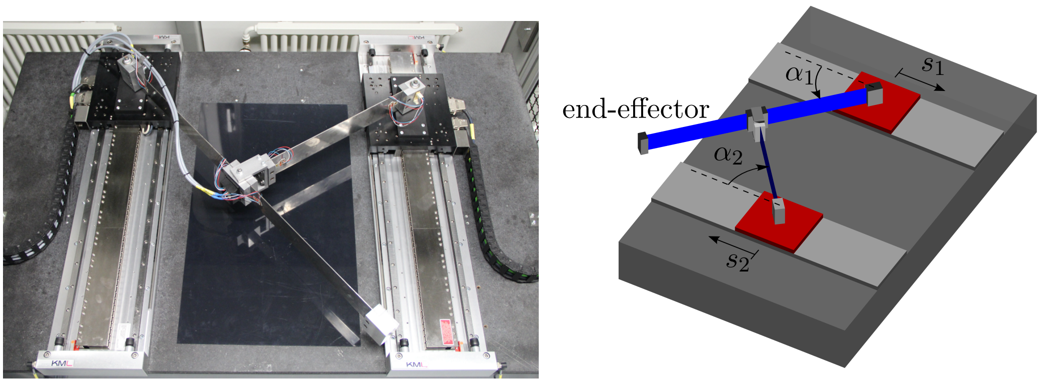
$$\boldsymbol{M}\ddot{\boldsymbol{q}} + \boldsymbol{k}(\dot{\boldsymbol{q}}, \boldsymbol{q}) = \boldsymbol{g}(\dot{\boldsymbol{q}}, \boldsymbol{q}) + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{C}^T\boldsymbol{\lambda}\,, \tag{1a}$$

$$\boldsymbol{c}(\boldsymbol{q}) = \boldsymbol{0}\,. \tag{1b}$$

The mass matrix of the flexible multibody system $\boldsymbol{M} \in \mathbb{R}^{n\times n}$ is symmetric, positive definite and depends on the joint angles and the elastic coordinates. The vector $\boldsymbol{k} \in \mathbb{R}^n$ contains the generalized centrifugal, Coriolis and Euler forces and $\boldsymbol{g} \in \mathbb{R}^n$ includes the vector of applied forces and inner forces due to the body elasticity. The input matrix $\boldsymbol{B} \in \mathbb{R}^{n\times p}$ maps the input vector $\boldsymbol{u} \in \mathbb{R}^p$ to the system. The constraint equations are shown by $\boldsymbol{c} \in \mathbb{R}^q$, a Jacobian matrix of constraint is shown by $\boldsymbol{C} \in \mathbb{R}^{n\times q}$ and $\boldsymbol{\lambda} \in \mathbb{R}^q$ is the reaction force during kinematic loop. Based on the projection of the constraint Jacobian matrix, the term $\boldsymbol{C}^T\boldsymbol{\lambda}$ can be removed [12]. The equation of motion is rewritten as follow

$$\overline{\boldsymbol{M}}\ddot{\boldsymbol{q}} + \overline{\boldsymbol{k}}(\dot{\boldsymbol{q}}, \boldsymbol{q}) = \overline{\boldsymbol{g}}(\dot{\boldsymbol{q}}, \boldsymbol{q}) + \overline{\boldsymbol{B}}\boldsymbol{u}. \tag{2}$$

Figure 1b shows the modeling result of the flexible parallel lambda robot in *Neweul-M*^{2}. The model of the flexible parallel lambda robot includes two active prismatic joints (s_1, s_2) for which control inputs are calculated, two passive revelute joints (α_1, α_2) and some elastic coordinates.



(a) Mechanical setup of the lambda manipulator **(b)** Lambda model in *Neweul-M*^{2}

Figure 1. Lambda robot in hardware and simulation

2 Nonlinear Observer

A nonlinear system in state space is presented as follow

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{H}(\boldsymbol{x})\boldsymbol{u}\,, \tag{3a}$$

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}\,, \tag{3b}$$

where $\boldsymbol{x} \in \mathbb{R}^{2n}$, $\boldsymbol{y} \in \mathbb{R}^{m}$ and $\boldsymbol{u} \in \mathbb{R}^{p}$ are the states, outputs, and inputs of system that is defined as $\boldsymbol{u} = \boldsymbol{K}\boldsymbol{x}$ with constant matrix gain $\boldsymbol{K} \in \mathbb{R}^{p\times 2n}$. The number of actuated joint of flexible multibody system is defined with p which for the lambda robot is equal to 2. Also, $\boldsymbol{g}(\boldsymbol{x}) \in \mathbb{R}^{2n}$, $\boldsymbol{H} \in \mathbb{R}^{2n\times p}$, $\boldsymbol{A} \in \mathbb{R}^{2n\times 2n}$, and $\boldsymbol{C} \in \mathbb{R}^{m\times 2n}$ respectively are the continuous nonlinear functions of system states, constant gain of the states, and the output gain matrices. The nonlinear functions of states $\boldsymbol{f}(\boldsymbol{x}) \in \mathbb{R}^{2n}$ is defined as follow

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{H}(\boldsymbol{x})\boldsymbol{u} = \boldsymbol{g}(\boldsymbol{x}) + \boldsymbol{H}(\boldsymbol{x})\boldsymbol{K}\boldsymbol{x}. \tag{4}$$

The problem of the states estimation $\boldsymbol{x}$ for the system in Eq. 3 is usually referred to design a high gain nonlinear observer [6, 10]. Here, the formulation of the observer states is written as

$$\dot{\hat{\boldsymbol{x}}} = \boldsymbol{A}\hat{\boldsymbol{x}} + \boldsymbol{f}(\hat{\boldsymbol{x}}) + \boldsymbol{L}(\hat{\boldsymbol{y}} - \boldsymbol{y})\,, \tag{5a}$$
$$\hat{\boldsymbol{y}} = \boldsymbol{C}\hat{\boldsymbol{x}}\,, \tag{5b}$$

where $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ are the estimated states and outputs of the observed system. Therefore, the observer gain $\boldsymbol{L} \in \mathbb{R}^{2n\times q}$ should be designed somehow such that the observed nonlinear system converges to the real system. The state observation error is different between the estimation and real system states, which is calculated by $\boldsymbol{e} = \hat{\boldsymbol{x}} - \boldsymbol{x}$. Hence, the dynamics of observed error is

$$\dot{\boldsymbol{e}} = (\boldsymbol{A} + \boldsymbol{L}\boldsymbol{C})\boldsymbol{e} + (\boldsymbol{f}(\boldsymbol{x} + \boldsymbol{e}) - \boldsymbol{f}(\boldsymbol{x})). \tag{6}$$

If the nonlinear dynamic error in Eq. 6 converges asymptotically to zero, it can be concluded that the estimation state converges to real system states. To show the estimation error converges to zero, the Lyapunov method and Lipschitz condition are used. The Lyapunov candidate function and its derivative are defined as

$$V(\boldsymbol{e}) = \boldsymbol{e}^T\boldsymbol{P}\boldsymbol{e}, \tag{7}$$

$$\dot{V}(\boldsymbol{e}) = \boldsymbol{e}^T((\boldsymbol{A} + \boldsymbol{L}\boldsymbol{C})^T\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} + \boldsymbol{L}\boldsymbol{C}))\boldsymbol{e} + (\boldsymbol{f}(\hat{\boldsymbol{x}}) - \boldsymbol{f}(\boldsymbol{x}))^T\boldsymbol{P}\boldsymbol{e} + \boldsymbol{e}^T\boldsymbol{P}(\boldsymbol{f}(\hat{\boldsymbol{x}}) - \boldsymbol{f}(\boldsymbol{x})). \tag{8}$$

The Lyapanov candidate is defined as positive-definite function for the estimation error as Eq. 7 with an unique positive definite matrix $\boldsymbol{P} \in \mathbb{R}^{2n\times 2n}$. The derivative of the Lyapunov function should be negative to ensure the estimation error converges asymptotically to zero. Toward this goal, an additional constraint is required. The nonlinear function $\boldsymbol{f}(\boldsymbol{x})$ should satisfy the Lipschitz condition, too. Also, there exists a positive definite matrix $\boldsymbol{Q} \in \mathbb{R}^{2n\times 2n}$ that is defined from

$$(\boldsymbol{A} + \boldsymbol{L}\boldsymbol{C})^T\boldsymbol{P} + \boldsymbol{P}(\boldsymbol{A} + \boldsymbol{L}\boldsymbol{C}) = -2\boldsymbol{Q}. \tag{9}$$

Based on the Lipschitz condition, there exists a constant G such that

$$\|\boldsymbol{f}(\hat{\boldsymbol{x}}) - \boldsymbol{f}(\boldsymbol{x})\| \leqslant G\|\hat{\boldsymbol{x}} - \boldsymbol{x}\|, \quad \forall i = 1, .., n \tag{10}$$

for all $\boldsymbol{x}$ and $\hat{\boldsymbol{x}} \in \mathbb{R}^{2n}$. Therefore, for Eq. 8, the following inequalities are valid.

$$\dot{V}(\boldsymbol{e}) \leqslant -2\boldsymbol{e}^T\boldsymbol{Q}\boldsymbol{e} + 2G\|\boldsymbol{P}\boldsymbol{e}\|\|\boldsymbol{e}\| \leqslant (-2\boldsymbol{\sigma}_{\min}(\boldsymbol{Q}) + 2G\boldsymbol{\sigma}_{\max}(\boldsymbol{P}))\|\boldsymbol{e}\|^2. \tag{11}$$

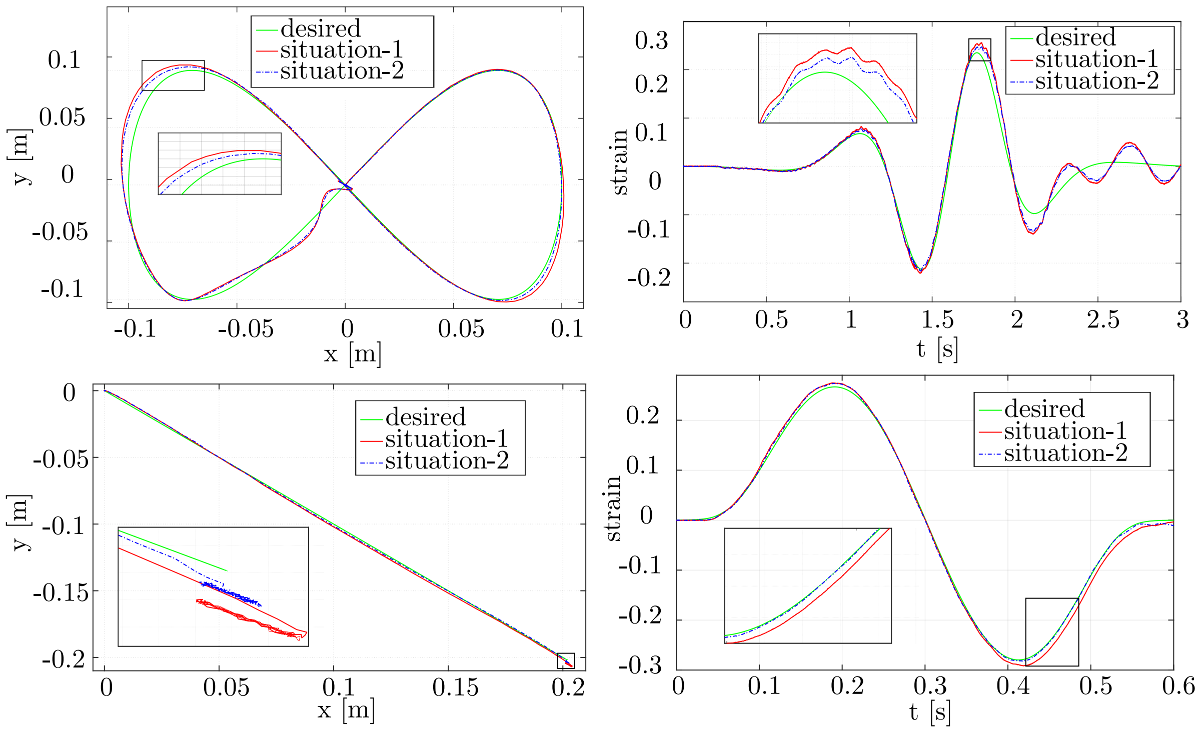
Here, $\boldsymbol{\sigma}_{\min}(\boldsymbol{Q})$ is the minimum singular value of the matrix $\boldsymbol{Q}$ and $\boldsymbol{\sigma}_{\max}(\boldsymbol{P})$ is the maximum singular value of the matrix $\boldsymbol{P}$. In order to have a negative derivative of the Lyapunov function, this condition should be satisfied for matrices $\boldsymbol{Q}$ and $\boldsymbol{P}$.

$$\frac{\boldsymbol{\sigma}_{\min}(\boldsymbol{Q})}{\boldsymbol{\sigma}_{\max}(\boldsymbol{P})} \leqslant G \tag{12}$$

Finally, the above conditions show the estimation error asymptotically converge to zero. The observer gain matrix $\boldsymbol{L}$ and unique matrix $\boldsymbol{P}$ are provided by satisfying Eqs. 9, 10 and 12.

3 Experimental Results

The designed observer is used to the model improvement, states, and end-effector position estimation. In order to compare two models and validate the estimation results for the position of the end-effector and the elastic coordinates, a camera and a strain gauge are used. The camera tracks the end-effector position off-line. Also, to validate the estimation results of elastic coordinates, the deformation of a strain gauge and observer for the long link are compared.

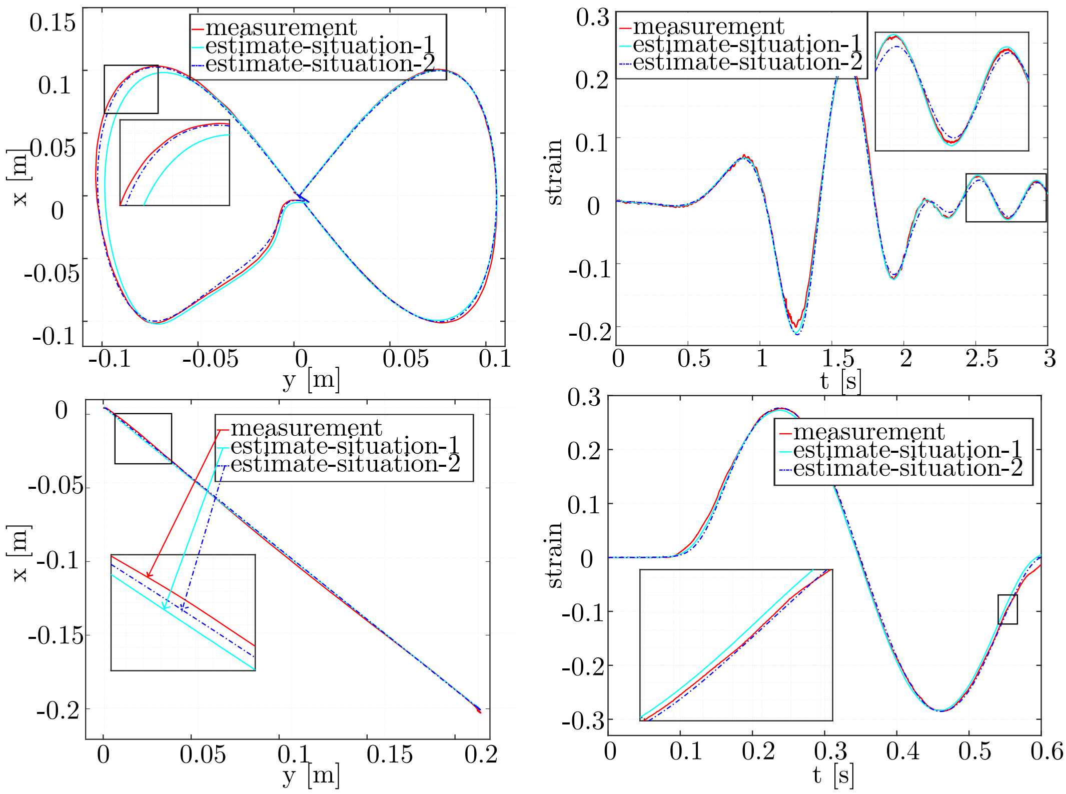


(a) End-effector position **(b)** Strain of long link

Figure 2. Experimental comparison of two model for nonlinear and linear trajectories

The nonlinear observer design conditions in Eqs. 9, 10 and 12 show the observed system states converge to the real system states. Based on this, the observed position must converge to the measurement position of the end-effector. While based on the model situation-1 in [2], the observed positions did not converge to the measurement positions of the end-effector. To solve this problem, the modeling parameters are observed. Based on the observation and measurement height and stiffness of the flexible beam is decreased. Then, the model of the flexible parallel manipulator is improved which are shown in Figure 2 as a situation-2. The experimental results show that the model situation-2 decreases the maximum and oscillation error for the linear and nonlinear trajectories.

The results of observer estimation in online processing are shown in Figure 3. The estimations of the elastic coordinates of elastic deformations of the long link track are as accurate as the strain gauge measurements. Comparing the off-line camera measurements and the online estimation results, demonstrates the accuracy of the proposed observer based on the model situation-2. This shows the position of the end-effector and the elastic states converge to the real value. The observer based on the situation-2 of model estimates the end-effector positions with an accuracy of about 1 millimeter maximum absolute error for a linear and about 2.5 millimeter maximum absolute error for a nonlinear trajectory.



(a) End-effector position **(b)** Strain of long link

Figure 3. Compare measurement results of the camera and strain gauge with observer results based on the model situation-1 and situation-2 for the linear and nonlinear trajectories

Conclusions

In this contribution, a nonlinear observer was designed and applied experimentally to a very flexible multibody system. The nonlinear observer is designed based on the position and velocity of prismatic joints and only the deformation of the long link. Also, the stability and convergence of the dynamic error of the estimation states based on the Lyapunov candidate function was shown. The experimental results for the very flexible parallel robot showed that the observer successfully tracks the measurements. Also, the observer estimates the nonlinear system states and end-effector positions with high accuracy.

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