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Fiber-Orientation-Evolution Models for Compression Molding of Fiber Reinforced Polymers

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Micro Abstract
This presentation gives a short review on the Fiber-Orientation-Evolution models commonly used in commercial softwares. The basis of the considered models is Jeffery's equation from 1922 which describes the motion of a single ellipsoidal particle in a Newtonian fluid. The later models extend Jeffery's equation for many fiber system, with the use of Fiber-Orientation-Tensors. Models are discussed up to the o-iARD-RPR model (Tseng 2016), and compared considering representative flow modes.

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Introduction

The anisotropic mechanical properties of a discontinuous fiber reinforced polymer part mainly depend on the fiber orientation distribution. To forecast the direction dependent properties, the production process has to be simulated which is commonly injection- or compression molding. The next session gives an overview of the several, so-called fiber orientation evolution models which were developed in the last few decades. Two of these models are implemented and compared considering representative flow modes and different closure approximations. Through the Abaqus VUMAT (Vectorized User Material) interface the Folgar-Tucker model with the quadratic closure is implemented and applied for the compression molding of a plate.

Model equations

Jeffery's equation

The foundation for all considered fiber orientation evolution models is Jeffery's equation [6] which describes the motion of a single ellipsoidal particle in a Newtonian fluid

$$\dot{\boldsymbol{n}}_\alpha = \bar{\boldsymbol{W}}[\boldsymbol{n}_\alpha] + \bar{\xi}(\bar{\boldsymbol{D}}[\boldsymbol{n}_\alpha] - (\boldsymbol{n}_\alpha \otimes \boldsymbol{n}_\alpha \otimes \boldsymbol{n}_\alpha)[\bar{\boldsymbol{D}}]) \quad , \quad \bar{\xi} = \frac{\bar{a}^2 - 1}{\bar{a}^2 + 1} \; , \; \bar{a} = \bar{l}/\bar{d}. \tag{1}$$

The normalized orientation vector of a fiber α is noted with $\boldsymbol{n}_\alpha$ and $\dot{\boldsymbol{n}}_\alpha$ indicates the material derivative of it. The effective vorticity and rate-of-deformation tensors are $\bar{\boldsymbol{W}}$ and $\bar{\boldsymbol{D}}$, respectively. The geometry factor $\bar{\xi}$ is defined through the aspect ratio $\bar{a}$ of a representative fiber with the length $\bar{l}$ and diameter $\bar{d}$. The dyadic product is noted with $\otimes$ and a linear mapping with [ ]. A modern derivation of Jeffery's equation can be found in [7].

Numerically it is more efficient to use fiber orientation tensors [1] instead of modeling each of the fibers. The empirical, reducible second and fourth order fiber orientation tensors of the first kind [8], belonging to M pieces of fibers, are defined as

$$\bar{\boldsymbol{N}} = \frac{1}{M}\sum_{\alpha=1}^{M} \boldsymbol{n}_\alpha \otimes \boldsymbol{n}_\alpha \quad , \quad \bar{\mathbb{N}} = \frac{1}{M}\sum_{\alpha=1}^{M} \boldsymbol{n}_\alpha \otimes \boldsymbol{n}_\alpha \otimes \boldsymbol{n}_\alpha \otimes \boldsymbol{n}_\alpha. \tag{2}$$

With the use of these fiber orientation tensors Jeffery's equation can also be written in a tensorial form

$$\dot{\bar{\boldsymbol{N}}}^{\mathrm{h}} = \bar{\boldsymbol{W}}\bar{\boldsymbol{N}} - \bar{\boldsymbol{N}}\bar{\boldsymbol{W}} + \bar{\xi}(\bar{\boldsymbol{D}}\bar{\boldsymbol{N}} + \bar{\boldsymbol{N}}\bar{\boldsymbol{D}} - 2\bar{\mathbb{N}}^{*}[\bar{\boldsymbol{D}}]). \tag{3}$$

The upper index h indicates that (3) describes the reorientation for the case that only hydrodynamic forces are acting. The forth order fiber orientation tensor $\bar{\mathbb{N}}$ is marked with a $*$ in (3), because different approximations can be used to calculate it. The simplest one is the quadratic closure [4]

$$\bar{\mathbb{N}}^{\mathrm{Quad}} = \bar{\boldsymbol{N}} \otimes \bar{\boldsymbol{N}}. \tag{4}$$

This formula is only exact for unidirectional fiber orientation distributions. For more precise calculations, in this paper, the Invariant Based Optimal Fitting (IBOF) closure [3]

$$\begin{aligned} \bar{\mathbb{N}}^{\mathrm{IBOF}} = \; & \bar{\beta}_1 \,\mathrm{sym}(\mathbf{1} \otimes \mathbf{1}) + \bar{\beta}_2 \,\mathrm{sym}(\mathbf{1} \otimes \bar{\boldsymbol{N}}) + \bar{\beta}_3 \,\mathrm{sym}(\bar{\boldsymbol{N}} \otimes \bar{\boldsymbol{N}}) + \\ & \bar{\beta}_4 \,\mathrm{sym}(\mathbf{1} \otimes \bar{\boldsymbol{N}}\bar{\boldsymbol{N}}) + \bar{\beta}_5 \,\mathrm{sym}(\bar{\boldsymbol{N}} \otimes \bar{\boldsymbol{N}}\bar{\boldsymbol{N}}) + \bar{\beta}_6 \,\mathrm{sym}(\bar{\boldsymbol{N}}\bar{\boldsymbol{N}} \otimes \bar{\boldsymbol{N}}\bar{\boldsymbol{N}}) \end{aligned} \tag{5}$$

is used. The rule, how to calculate $\bar{\beta}_i,\ i = 1 \dots 6$, as a function of the second and third invariants of $\bar{\boldsymbol{N}}$, is given in [3]. The sym( ) operator calculates the symmetric part of its argument and $\mathbf{1}$ notes the second order identity tensor. There are several other types of closure approximations which are not considered in this paper.

Extensions of Jeffery's equations

To consider the effect of the fiber-fiber interaction Folgar and Tucker (FT) [5] added an isotropic rotary diffusion term to (3)

$$\dot{\bar{\boldsymbol{N}}}^{\mathrm{FT}} = \dot{\bar{\boldsymbol{N}}}^{\mathrm{h}} + 2\bar{C}_{\mathrm{I}}\dot{\bar{\gamma}}(\boldsymbol{I} - 3\bar{\boldsymbol{N}}) \quad , \quad \dot{\bar{\gamma}} = \sqrt{(2(\bar{\boldsymbol{D}} \cdot \bar{\boldsymbol{D}}))}\ ,\ \bar{C}_{\mathrm{I}} = \bar{f}(\bar{c}_{\mathrm{f}}). \tag{6}$$

The FT model predicts to fast fiber reorientation in comparison to experiments. To slow down the fiber reorientation Wang et al. introduced the Reduced Strain Closure (RSC) model [11]. To be able to predict Anisotropic Rotary Diffusion (ARD) Phelps and Tucker introduced the ARD model [9] which can be combined with the RSC model. The ARD-RSC model includes 6 phenomenological parameters. To reduce the number of these parameters Tseng et al. introduced the objective improved ARD model with Retarding Principal Rate (o-iARD-RPR) [10] which contains only 4 phenomenological parameters.

Numerical implementation and results

Representative flow modes

The tensorial form of Jeffery's equation (3) and the Folgar-Tucker model (6) are implemented with the quadratic (4) and IBOF closures and compared for representative flow cases. The investigated shear, elongational, compressional and planar flow cases are the same as described in [2]. The initial fiber orientation distribution is isotropic. The similarities and differences of the four investigated models are represented here on the example of the $\bar{N}_{\mathrm{xx}}$ component vs. the von Mises equivalent strain ε_{M}, considering the shear (Figure 1a)) and the elongational (Figure 1b)) flow.

2D Eulerian press model

A fluid material model is implemented through the VUMAT interface. This fluid model is tested for simple shear, elongation and compression, with the help of a one element Lagrangian model, and it is proven that the numerical results match the analytical solutions.

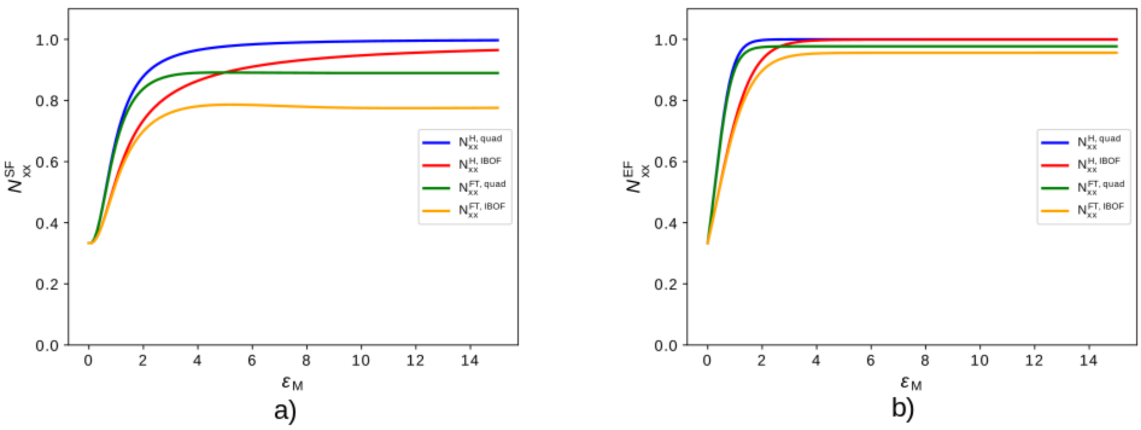

Figure 1. The evolution of the the $\bar{N}_{xx}$ component in a) shear flow and b) in elongational flow

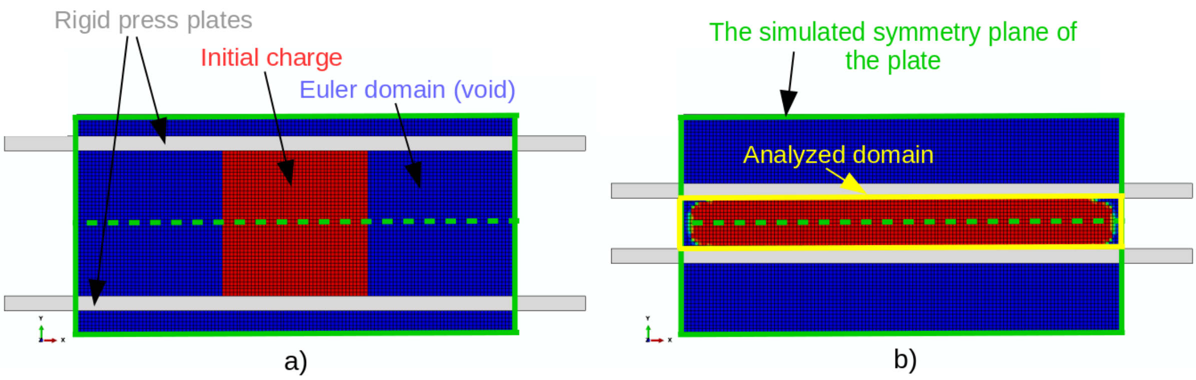

Figure 2. 2D Eulerian press model in Abaqus, a) initial state, before compression, b) end state, after compression

For the 2D Eulerian press model the Folgar-Tucker equation (6) with the quadratic closure (4) are implemented. The 2D, Eulerian press model is depicted in Figure 2.

The initial fiber orientation distribution is planar isotropic in the x-y plane. The final spatial distribution of the diagonal components of $\bar{\boldsymbol{N}}$ are represented in Figure 3.

Conclusions

With the introduced 2D Eulerian press model the shell-core-shell fiber orientation layers can be analyzed, and all of the constitutive equations can be modified. The shell layer is not modeled correctly yet, because the actual model does not consider the high temperature gradients at the contact surface of the press plate and the compressed material. The model can be developed to investigate the effect of the anisotropic effective viscosity [2].

Acknowledgements

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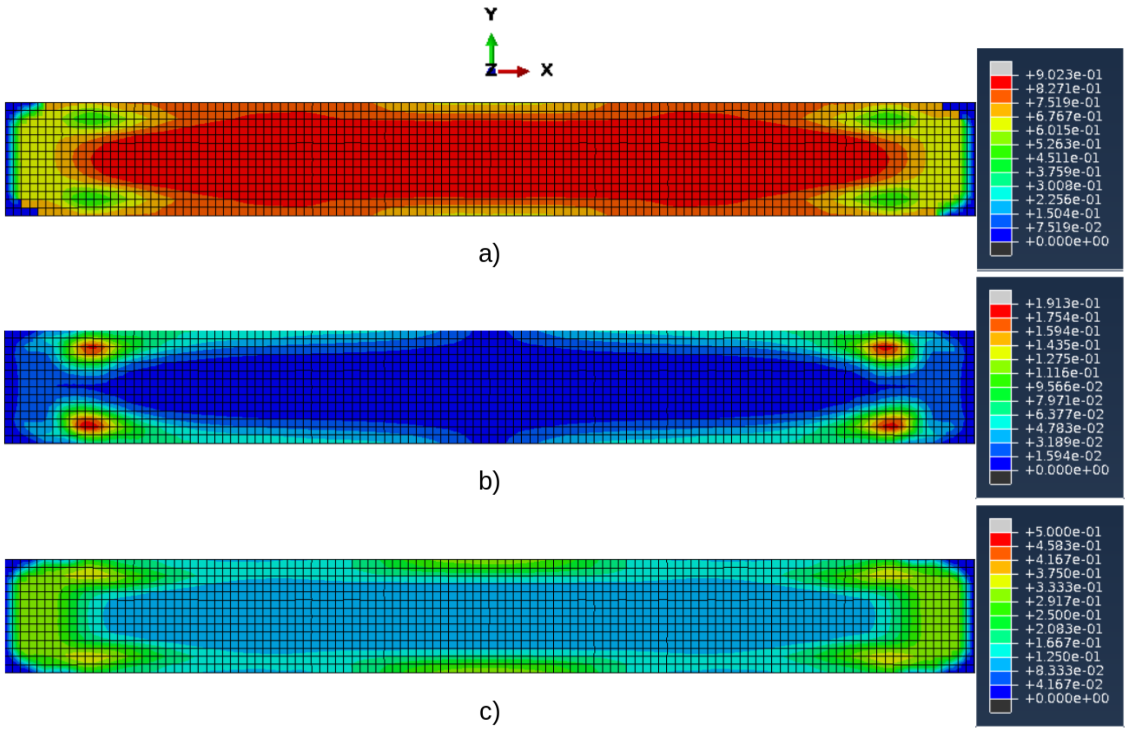

Figure 3. Spatial distribution of the diagonal components of $\bar{\boldsymbol{N}}$ in the analyzed domain, a) $\bar{N}_{\mathrm{xx}}$ component, b) $\bar{N}_{\mathrm{yy}}$ component, c) $\bar{N}_{\mathrm{zz}}$ component

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