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Variable filter radii for Vertex Morphing based design of adaptive structures

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Micro Abstract
Most optimization tasks on adaptive structures focus on the optimal actuator/sensor placement. Also, the shape of the controlled structure plays an important role for its efficiency. The initial applications of Vertex Morphing, a node based shape optimization method, made use of a single filter radius. This contribution will assess the effect of varying filter radii towards a better control of the shape, whereas the rich design space given with the Vertex Morphing method shall be maintained.

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Introduction

Many shape adaptive shell structures use hinge-like bellow zones in order to maximize the displacement resulting from an actuation. Finding the actual shape of those zones is a challenging task and crucial for the performance of the actuated shell structure. Node based shape optimization methods in combination with adjoint sensitivity analysis have been successfully used to optimize complex structures and can offer design ideas beyond engineering intuition [2, 3, 5]. In this contribution, a varying filter radius for shape optimization using the Vertex Morphing Method is investigated. In context of node based shape optimization, similar ideas have been followed by Masching [4, 5], who used a single filter radius for all shape variables and another radius for the filtering of nodal actuation forces in a structural optimization of an adaptive structure. Bletzinger [1] has gradually reduced the filter radius towards the border of the design space in order to interpolate the control field at the borders of one dimensional regular grids.

Vertex Morphing

Vertex Morphing is a well established method for node-based shape optimization. The geometry $z(x_i)$ at a position x_i on the surface is created by convoluting a filter function F with a control field $p(x)$.

$$z(x_i) = \int_{\Gamma} F(x, x_i, r) p(x) d\Gamma \tag{1}$$

In a gradient based shape optimization, the derivative of the response function $f(x, z(x, p))$ with respect to the geometry z may be calculated using an adjoint sensitivity analysis. The derivative of the response function with respect to the control parameters p can be straight forwardly derived from equation 1:

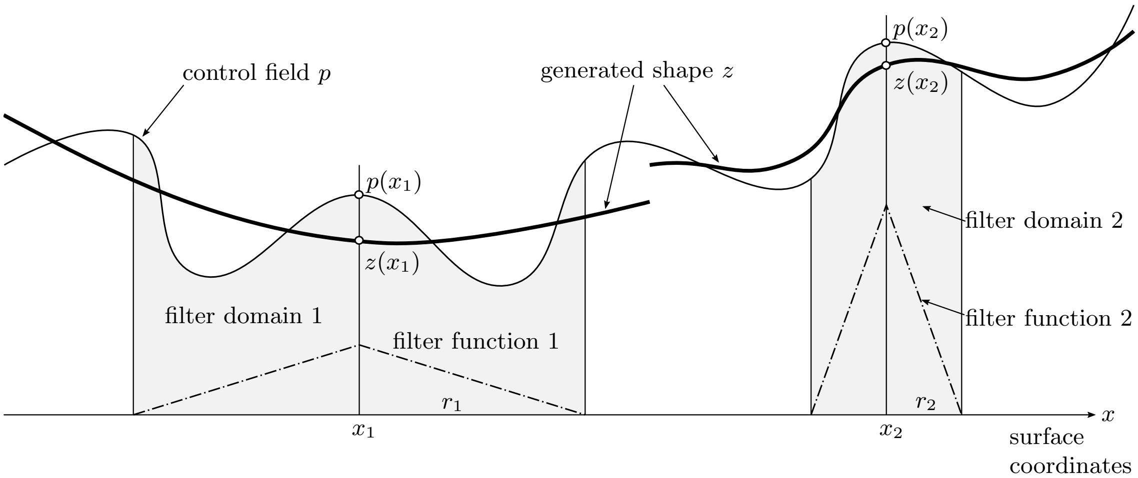

Figure 1. Effect of varying filter radius r_i on the shape z generated from a given control field p.

$$\frac{dz(x_i)}{dp(x_j)} = F(x_j, x_i, r) \tag{2}$$

$$\left.\frac{df}{dp}\right|_{x_i} = \int_\Gamma \frac{df}{dz}\frac{dz}{dp(x_i)}d\Gamma = \int_\Gamma \frac{df}{dz}F(x_i, x, r)d\Gamma \tag{3}$$

It is important to note that the control field p does not need to be known, the Vertex Morphing Method works with relative changes of the control field and shape, respectively.

If the same discretization is used for the geometry z and the control field p the modelling effort is minimized, because the computational mesh can be directly used for the optimization. The interested reader is referred to [1, 3] for a detailed derivation.

In the original formulation of the Vertex Morphing Method from [1, 3], a single filter radius r was used for the whole domain, making it the only decisive parameter for the setup of the shape optimization problem. This single design handle r guides the optimization towards a local optimum characterized by a lengthscale of the shape corresponding to r.

Variable Filter Radius

The single design handle r, as described in the previous section, minimizes the required input from the designer, and allows for the fastest possible setup of the optimization process. Nevertheless, it might be necessary to have more control on the shape parametrization depending on the location on the surface. A variable filter radius in space is suggested and investigated in the following.

The geometry $z(x_i)$ at a position x_i is still generated by a local filter operation, but now with filter properties specific to the surface coordinate, e.g. the filter radius $r(x_i)$ corresponding to a surface coordinate x_i.

$$z(x_i) = \int_\Gamma F(x, x_i, r(x_i))p(x)d\Gamma \tag{4}$$

Like in the previous section, shape derivative and mapping of the sensitivities can be derived

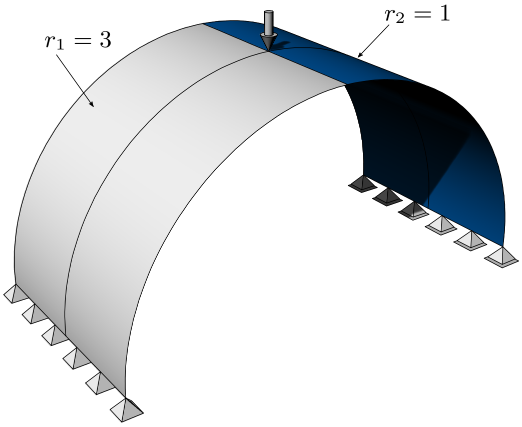

Figure 2. Simple shape optimization example of a shell with two filter radii r_1 and r_2.

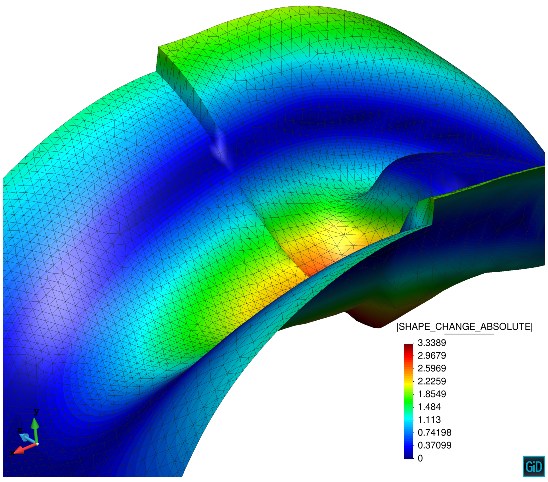

Figure 3. Undesired jump in shape update if the filter radius changes abruptly.

straight forwardly from equation 4:

$$\frac{dz(x_i)}{dp(x_j)} = F(x_j, x_i, r(x_i)) \tag{5}$$

$$\left.\frac{df}{dp}\right|_{x_i} = \int\limits_{\Gamma} \frac{df}{dz}\frac{dz}{dp(x_i)}d\Gamma = \int\limits_{\Gamma} \frac{df}{dz}F(x_i, x, r(x))d\Gamma \tag{6}$$

As the definition of a complete field of filter radii would require too much input from the designer, in the following only two main radii will be further investigated. In this way, the designer is able to prescribe areas of moderate shape changes using a large filter radius, and allow for shorter wavelength shape changes in other areas.

Transition between different radii

A sudden jump in size of the filter radius $r(x) \gg r(x + dx)$ results in completely different integration domains for the description of two neighboring surface coordinates according to equation 4. In general this leads to an undesired jump in the resulting shape. This can easily be seen in figures 1 and 3.

As a remedy, a transition zone with a blending of the two neighbouring filter radii, similar to [1], is proposed. The extension of the transition zone and the shape of the blending function $\xi(x)$ with $0 \leq \xi \leq 1$ can be chosen arbitrarily.

$$r(x_i) = \xi(x_i)r_1 + (1 - \xi(x_i))r_2 \tag{7}$$

Another option is the blending of the resulting geometry which can also be seen as an overlapping of the two filter functions using the same blending function as above.

$$z(x_i) = \xi(x_i)\underbrace{\int\limits_{\Gamma} F_1(x, x_i, r_1)p(x)d\Gamma}_{z_1} + (1 - \xi(x_i))\underbrace{\int\limits_{\Gamma} F_2(x, x_i, r_2)p(x)d\Gamma}_{z_2} \tag{8}$$

If one part of the equation 8 e.g. z_2 is set to a constant coordinate, it can be used to blend from a non design space part smoothly into the shape update. This is often desired at the boundaries

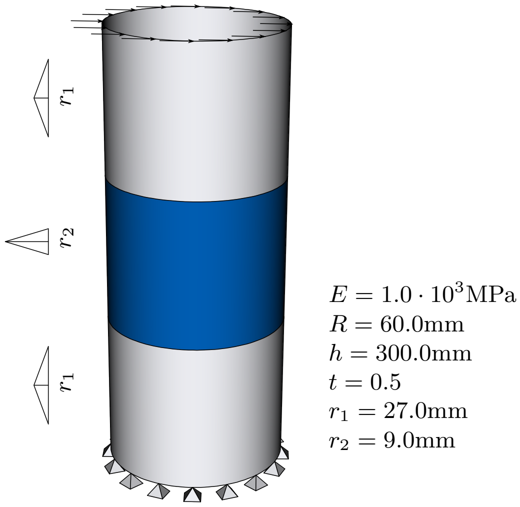

Figure 4. Setup of the shape optimization with two filter radii r_1 and r_2.

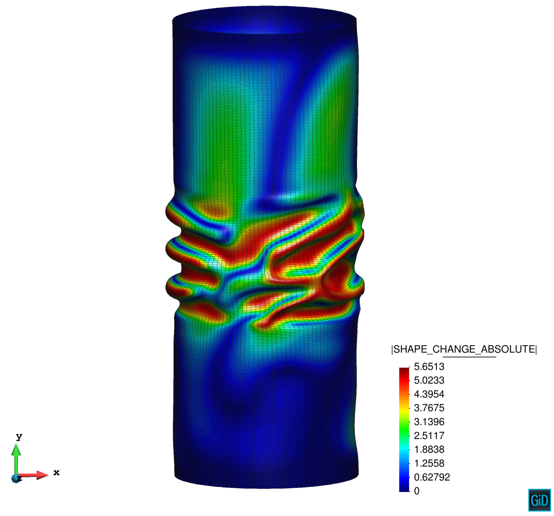

Figure 5. Resulting shape with small bellows in the middle and moderate shape changes at top and bottom.

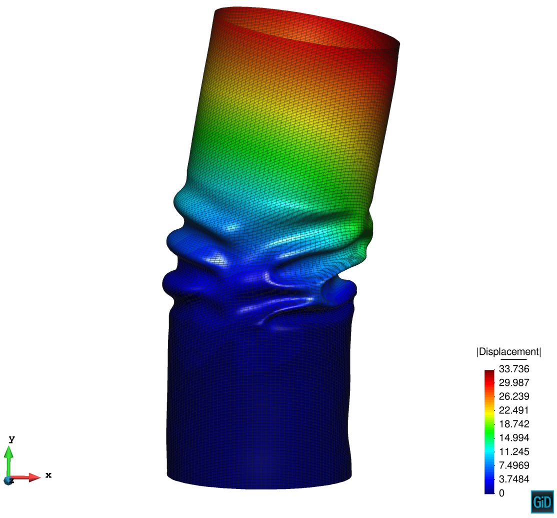

Figure 6. Deformation of the optimized and loaded shape.

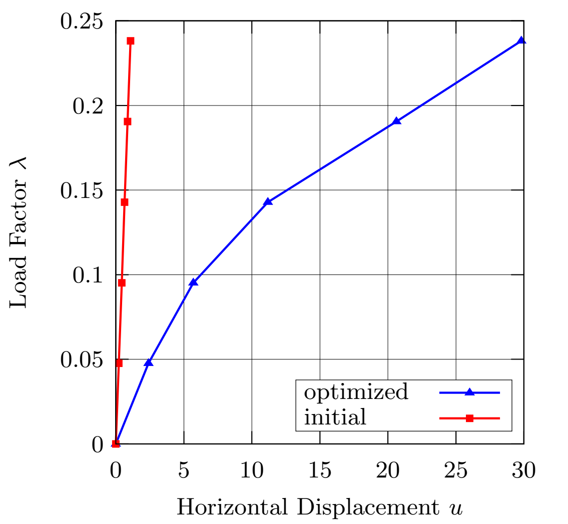

Figure 7. Load displacement curve of the optimized and the initial structure.

of the design surface, or areas where boundary conditions are applied in the primal analysis. As an example, a cosine-blending function can be used to create a tangential transition at design surface boundaries.

Application Example

The Vertex Morphing Method with variable filter radii is applied in a shape optimization example of a bendable pipe structure. The pipe has a prescribed horizontal actuation at the top and is simply supported at the bottom. The objective is to increase the horizontal displacement at the top of the pipe. The hinging zone is desired to be in the blue area in figure 4. Hence a small filter radius $r_2 = 9$ is used in that area, compared to the rest of the pipe with $r_1 = 27$. A geometrically non-linear displacement response function [4] is maximized, while the bead depth is constrained. The optimization results in a shape with a bellow zone of the expected wave length in the middle, and moderate shape changes at the top and bottom, as can be seen in figure 5.

Conclusions

Variable filter radii for the Vertex Morphing Method have been investigated and succesfully applied in a shape optimization of the passive part of an adaptive shell structure. This extension of the method allows to define desired regions for bellow-like hinging zones, whereas the rest of the structure can still be optimized with more moderate shape changes.

Acknowledgements

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