Proceedings of the 7th GACM Colloquium on Computational Mechanics
for Young Scientists from Academia and Industry
October 11-13, 2017 in Stuttgart, Germany

General imperfect interface models at finite deformations

Tim Heitbreder[1]* and Jörn Mosler[1]

Micro Abstract
Considering a geometrically exact description, only isotropic classical cohesive zone models fulfill fundamental principles such as material frame indifference and thermodynamical consistency. The ability to model shear and anisotropy is limited. Within this talk, a novel interface model, which is consistent with the above mentioned fundamental principles, is presented. Besides the simulation of anisotropic hyperelasticity, numerical results for anisotropic material degradation are shown.

[1]Institute of Mechanics, TU Dortmund University, Dortmund, Germany
***Corresponding author**: tim.heitbreder@tu-dortmund.de

Introduction

A possible classification of material interfaces is to distinguish between so-called *coherent interfaces*, that show a discontinuous traction vector while the displacement field is continuous and *non-coherent interfaces*, where a displacement discontinuity is possible. A modeling framework falling into the range of non-coherent material interfaces is the *cohesive zone modeling* approach. In classical cohesive zone models, the traction vector is assumed as continuous across the interface. In an exact geometrical setting, it turns out that fundamental balance laws not easily are fulfilled for cohesive zone models. For instance, as shown in [4], balance of angular momentum requires that the traction vector is collinear with the displacement jump. It implies for elasticity that only isotropic models which may include isotropic damage fulfill balance of angular momentum. It was also demonstrated in [3] that the classical cohesive zone framework applied to anisotropic hyperelasticity leads to a non-physical non-vanishing dissipation. On this background, a novel non-standard extended cohesive zone framework accounting for elastoplastic deformations was recently proposed in [4]. A formulation which considers softening plasticity as well as an efficient numerical implementation of this framework was provided in [1]. Within this talk a further generalization in line with [5] is presented that allows to model a material anisotropy while simultaneously fulfilling all fundamental principles in material modeling . This generalization can be interpreted as a combination of *coherent* and *non-coherent interfaces*. Due to this combination, the novel framework enables the modeling of shear and anisotropic effects within the interface under the consideration of different fracture energies for the relevant fracture modes, i.e. mode-I and mode-II/III. Besides the theoretical derivation, numerical results for anisotropic hyperelasticity and material degradation are shown within this talk.

1 Interface kinematics

In the reference configuration, the interface Γ^0 under investigation, see Figure 1, is characterized by Cartesian coordinates $\boldsymbol{X} = \boldsymbol{X}(\xi_\beta)$, that are describes by curvilinear coordinates ξ_β, where the Greek index takes the values 1 and 2. The covariant tangential base vectors of the reference configuration, $\boldsymbol{G}_\beta$ are given by $\boldsymbol{G}_\beta = \dfrac{\partial \boldsymbol{X}}{\partial \xi_\beta}$. Furthermore, the unit normal base vector, that is

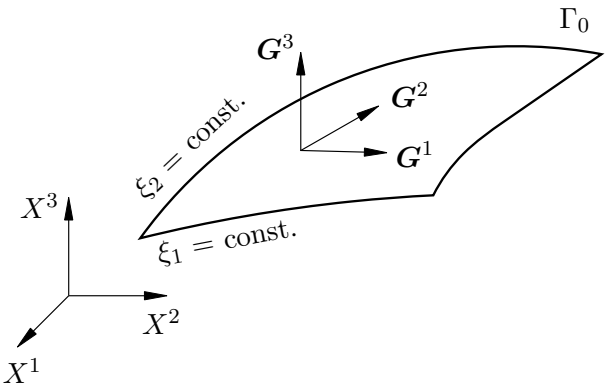

Figure 1. Illustration of the interface's reference configuration Γ_0 with the tangential base vectors $\boldsymbol{G}_\beta$ and the unit normal vector $\boldsymbol{G}_3$.

normal to the surface Γ_0 is calculated as

$$\boldsymbol{G}_3 = \frac{\boldsymbol{G}_1 \times \boldsymbol{G}_2}{|\boldsymbol{G}_1 \times \boldsymbol{G}_2|}. \tag{1}$$

Since this talk deals with non-coherent interfaces, discontinuities in the deformation field can occur. While in the reference configuration the upper surface Γ_{0+} and the lower surface Γ_{0-} of an interface coincide, i.e. $\Gamma_{0+} = \Gamma_{0-} = \Gamma_0$, the current configuration is characterized by different coordinates for the upper and the lower side of the opened interface, see Figure 2. The

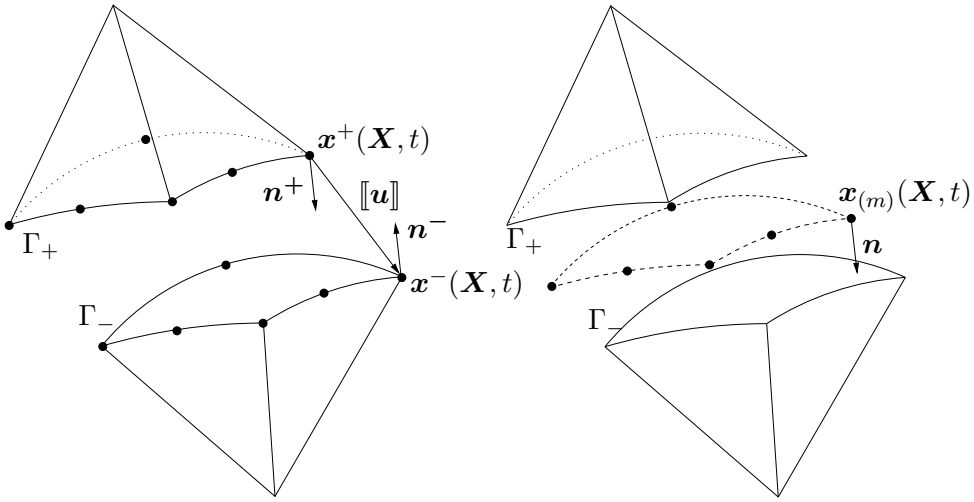

Figure 2. Opened material 12-node interface and fictitious intermediate configuration defining the structural tensors (dashed lines). The shape of the bulk is motivated by the finite element implementation - tetrahedron elements are employed.

displacement discontinuity $[\![\boldsymbol{u}]\!]$ is also known as the displacement jump which is defined as

$$[\![\boldsymbol{u}]\!] = \boldsymbol{x}^- - \boldsymbol{x}^+ = \boldsymbol{u}^- - \boldsymbol{u}^+. \tag{2}$$

Since the normal vector is not well defined in the deformed configuration for non-coherent interfaces, the so called mid-surface is introduced (dashed line in Fig. 2). A frequently made choice is the average of the upper and the lower coordinates, i.e.,

$$\boldsymbol{x}_{(m)} = \frac{1}{2}\left(\boldsymbol{x}^- + \boldsymbol{x}^+\right). \tag{3}$$

The normal vector of the current configuration with respect to this midsurface is given by

$$\boldsymbol{g}_3 = \frac{\boldsymbol{g}_1 \times \boldsymbol{g}_2}{|\boldsymbol{g}_1 \times \boldsymbol{g}_2|}, \tag{4}$$

where $\boldsymbol{g}_\beta$ denote the tangential base vectors with respect to the midsurface's current configuration. While the displacement jump describes the out-of-plane deformation of an interface the surface deformation gradient $\overline{\boldsymbol{F}}$ describes the in-plane deformation as a linear map of area elements between the reference and the current configuration, i.e.,

$$\overline{\boldsymbol{F}} = \boldsymbol{g}_\beta \otimes \boldsymbol{G}^\beta. \tag{5}$$

2 General imperfect interfaces

2.1 Hyperelasticity

For the consideration of anisotropic interface effects, a cohesive zone energy description just depending on the displacement jump $[\![\boldsymbol{u}]\!]$ is not sufficient. In order to distinguish between normal and shear separation according with mixed-mode loading, an extension of the isotropic framework is required. A possible extension of the classic cohesive zone framework is a definition of an interface energy depending on the displacement jump $[\![\boldsymbol{u}]\!]$ and the deformation dependent normal vector $\boldsymbol{n}$ of the current configuration. In this talk a further extension based on [5] is presented, where the interface energy is extended with the covariant tangential base vectors of the deformed configuration $\boldsymbol{g}_\beta$, i.e.,

$$\psi = \psi([\![\boldsymbol{u}]\!], \boldsymbol{g}_\beta). \tag{6}$$

This extension of the Helmholtz energy influences the corresponding stress power, i.e.,

$$\mathcal{P} = \overline{\boldsymbol{T}} \cdot \dot{[\![\boldsymbol{u}]\!]} + \boldsymbol{A}^\beta \cdot \dot{\boldsymbol{g}}_\beta. \tag{7}$$

Besides the average traction vector $\overline{\boldsymbol{T}}$ related to the rate of the displacement jump $\dot{[\![\boldsymbol{u}]\!]}$, additional stresses $\boldsymbol{A}^\beta$ related to the rate of the tangential base vectors $\boldsymbol{g}_\beta$ have to be considered. These additional tractions are related to membrane-like forces known from coherent interfaces and out-of-plane shear forces resulting from the non-coherency of the interface. The dissipation resulting from Eqs. (6) and (7) yields (for hyperelasticity)

$$\mathcal{D} = \dot{[\![\boldsymbol{u}]\!]} \cdot \left[\overline{\boldsymbol{T}} - \frac{\partial \psi^e}{\partial [\![\boldsymbol{u}]\!]}\right] + \dot{\boldsymbol{g}}_\beta \cdot \left[\boldsymbol{A}^\beta - \frac{\partial \psi^e}{\partial \boldsymbol{g}_\beta}\right] = 0, \tag{8}$$

where the classic Coleman and Noll procedure leads to

$$\overline{\boldsymbol{T}} = \frac{\partial \psi^e}{\partial [\![\boldsymbol{u}]\!]}; \quad \boldsymbol{A}^\beta = \frac{\partial \psi^e}{\partial \boldsymbol{g}_\beta}. \tag{9}$$

Due to the additional stresses $\boldsymbol{A}^\beta$ the traction vector no longer needs to be continuous, like it is the case in classical isotropic cohesive zone models, i.e. $\boldsymbol{T}^- \neq -\boldsymbol{T}^+$.

2.2 Extension with material degradation

In accordance with [3], [4] and [5] material degradation within the interface is realized through a scalar valued cross coupled damage model. To be more explicit, Helmholtz energy (6) is decomposed into a normal and a shear part, i.e.

$$\psi^e = \psi_n([\![\boldsymbol{u}]\!], \boldsymbol{g}_\beta) + \psi_s([\![\boldsymbol{u}]\!], \boldsymbol{g}_\beta). \tag{10}$$

Material degradation is captured by introducing a set of scalar-valued damage variables $d_i^{(j)}$. Each deformation mode is characterized by means of its own damage variable whereby an anisotropic failure behavior can be modeled. In order to couple damage in normal direction to that in shear direction, a mixed-mode energy of the type

$$\psi = (1 - d_n^{(n)})(1 - d_n^{(s)})\psi_n([\![\boldsymbol{u}]\!], \boldsymbol{g}_\beta) + (1 - d_s^{(n)})(1 - d_s^{(s)})\psi_s([\![\boldsymbol{u}]\!], \boldsymbol{g}_\beta) \tag{11}$$

is proposed.

3 Numerical studies of anisotropic damage

The talk is completed by numerical analyses of the influence of interfaces on the macroscopic response. Therefore, a cubic RVE with a spherical inclusion is investigated, cf. Figure 3. A macroscopic deformation gradient $\boldsymbol{F}$ is applied leading to debonding between the matrix and the inclusion. In order to compute the resulting macroscopic stresses, an extended homogenization scheme, introduced in [2], is applied. In addition to the purely hyperelastic interface, several

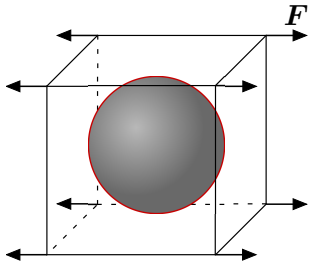

Figure 3. Numerical analysis of an RVE consisting of a spherical inclusion embedded in a matrix. The bulk materials are separated by a general imperfect interface. A macroscopic deformation gradient $\boldsymbol{F}$ is applied to the RVE

computations based on different damage models are also analyzed.

Conclusions

By elaborating an extended setting, it was shown that anisotropic cohesive zone models complying with all fundamentals in material modeling can be derived. The influence of the resulting advanced interface models on the macroscopic response was investigated through computational homogenization. It turns out that although all models involved are local in nature, their combination naturally captures a size effect due to the different scalling behavior (volume energies vs. interface energies).

References

[1] T. Heitbreder, N. S. Ottosen, M. Ristinmaa, and J. Mosler. Consistent elastoplastic cohesive zone model at finite deformations – variational formulation. *International Journal of Solids and Structures*, 106–107:284–293, 2017.

[2] A. Javili, P. Steinmann, and J. Mosler. Micro-to-macro transition accounting for general imperfect interaces. *Computer Methods in Applied Mechanics and Engineering*, 317:274–317, 2017.

[3] J. Mosler and I. Scheider. A thermodynamically and variationally consistent class of damage-type cohesive models. *Journal of the Mechanics and Physics of Solids*, 59:1647–1668, 2011.

[4] N. S. Ottosen, M. Ristinmaa, and J. Mosler. Fundamental physical principles and cohesive zone models at finite displacements - limitations and possibilities. *International Journal of Solids and Structures*, 53:70–79, 2015.

[5] N. S. Ottosen, M. Ristinmaa, and J. Mosler. Framework for non-coherent interface models at finite displacement jumps and finite strains. *Journal of the Mechanics and Physics of Solids*, 90:124–141, 2016.