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Phase-Field Modelling of Crack Propagation in Elasto-Plastic Multilayered Materials

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Micro Abstract
The phase-field method has emerged as an extremely powerful technique to simulate crack propagation with significant success. Phase-field simulation of crack propagation in elasto-plastic multilayered materials is discussed in this work. Three fundamental cases are studied i.e. (i) crack propagation in a brittle material, (ii) in a ductile material and (iii) in a brittle-ductile composite. The numerical results demonstrate the mechanical performance of such a multilayered composite design.

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Introduction

The phase-field method has emerged as an extremely powerful technique to simulate crack initiation and propagation with significant success [3]. Phase-field-type diffusive crack approach is capable of predicting the crack initiation and propagation without any additional criterion. Here, its main advantages are the ability to predict crack initiation and to handle curved crack paths, crack kinking, branching angles and crack-front segmentation in three dimensions. In this work, phase-field modelling of crack propagation in elasto-plastic multilayered materials is investigated. Therefore, we introduce the phase-field model for brittle fracture and phase-field model for ductile fracture which has been extended to the exhibiting J_2-plasticity material behavior. Three fundamental cases are studied i.e. (i) crack propagation in a brittle material, (ii) in a ductile material and (iii) in a brittle-ductile multilayered material. The numerical results demonstrate the mechanical performance of such a multilayered composite design. The multi-field coupled finite element problems are performed with staggered solutions.

1 Phase-field model of brittle fracture

In phase-model of brittle fracture, based on the regularized form of the variational model by Bourdin [1] the combined free energy density ψ is given as follows:

$$\psi(\boldsymbol{\varepsilon}, s) = (s^2 + \eta) \underbrace{\frac{1}{2}\boldsymbol{\varepsilon} : (\boldsymbol{C} : \boldsymbol{\varepsilon})}_{\psi_e} + \underbrace{G_c \gamma(s, \nabla s)}_{\psi_{frac}} \tag{1}$$

in which ψ_e is the elastic strain energy density of material with the total strain $\boldsymbol{\varepsilon}$ and the isotropic fourth-order elastic stiffness tensor $\boldsymbol{C}$. The phase-field variable s describes the intact state by $s = 1$ and the fully broken state of the material point is defined by $s = 0$. The small positive dimensionless parameter $0 < \eta \ll 1$ is used to ensure a numerically well-conditioned system for a fully-broken state ($s = 0$). The critical energy release G_c is defined as the energy required to create a unit area of new crack and $\gamma(s, \nabla s) = \frac{1}{4\kappa}(1 - s)^2 + \kappa|\nabla s|^2$ denotes the crack surface density function per unit volume of the solids. If κ tends towards zero, the phase-field approximation of the fracture energy density ψ_{frac} is exact. In order to ensure crack propagation under tensile or shear loading, a modified regularized formulation of the phase-field energy density was proposed in [3], using the definitions:

$$\psi(\boldsymbol{\varepsilon}, s) = (s^2 + \eta)\psi_e^+ + \psi_e^-(\boldsymbol{\varepsilon}) + G_c(\frac{1}{4\kappa}(1-s)^2 + \kappa|\nabla s|^2) \tag{2}$$

The elastic energy density ψ_e splits into the positive part ψ_e^+ and the negative part ψ_e^-, namely

$$\psi_e = \underbrace{\frac{1}{2}K(\langle tr(\boldsymbol{\varepsilon})\rangle^+)^2 + \mu(\boldsymbol{\varepsilon}^D : \boldsymbol{\varepsilon}^D)}_{\psi^+} + \underbrace{\frac{1}{2}K(\langle tr(\boldsymbol{\varepsilon})\rangle^-)^2}_{\psi^-} \tag{3}$$

where K is the bulk modulus, μ the shear modulus and $\boldsymbol{\varepsilon}^D$ the deviatoric part of the elastic strain $\boldsymbol{\varepsilon}$. Here, $\langle a\rangle^\pm = (a \pm |a|)/2$ define the so-called Macaulay bracket. A time dependent Ginzburg-Landau evolution equation of the crack phase-field s which is variational derived from the phase-field energy density reads as:

$$\frac{\dot{s}}{M} = -2s\psi_e^+ + G_c(\frac{1-s}{2\kappa} + 2\kappa\Delta s) \tag{4}$$

with the mobility factor M which should be chosen sufficiently large.

2 Phase-field model of ductile fracture

The energy density ψ is expressed as the sum of the elastic strain energy density, plastic strain energy density and the fracture energy density in the phase-field model for ductile fracture. The free energy density function ψ can be written as [2]:

$$\psi(\boldsymbol{\varepsilon}_e, s) = (s^2 + \eta)[\underbrace{\frac{1}{2}\boldsymbol{\varepsilon}_e : (\boldsymbol{C} : \boldsymbol{\varepsilon}_e)}_{\psi_e} + \underbrace{(\sigma_Y + \frac{1}{2}H\alpha)\alpha}_{\psi_p} - \psi_c] + \psi_c + \underbrace{\frac{4\kappa\psi_c}{\zeta}\gamma(s, \nabla s)}_{\psi_{frac}} \tag{5}$$

where ψ_e is the elastic energy density with the elastic strain tensor $\boldsymbol{\varepsilon}_e = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_p$. ψ_p is the plastic strain energy density assuming linear isotropic hardening with respect to the plastic strain tensor $\boldsymbol{\varepsilon}_p$, the accumulated plastic strain α, the yield stress σ_Y and the linear hardening coefficient H. The plastic strain tensor $\boldsymbol{\varepsilon}_p$ and the accumulated plastic strain α are defined as internal state variables within the considered J_2 plasticity model. ψ_c is a specific critical fracture energy per unit volume. ζ presents the post-critical range after crack initialization.
In order to distinguish between crack propagation in tension loadings, shear loadings and compression loadings, a modified regularized formulation of Eq. (5) was proposed in Miehe et al. [2],

$$\psi(\boldsymbol{\varepsilon}_e, \boldsymbol{\varepsilon}_p, \alpha, s) = (s^2 + \eta)[\psi_e^+(\boldsymbol{\varepsilon}_e) + \psi_p(\boldsymbol{\varepsilon}_p, \alpha) - \psi_c] + \psi_e^-(\boldsymbol{\varepsilon}_e) + \psi_c + \psi_{frac}(s, \nabla s) \tag{6}$$

which contains the decomposition of the elastic energy density $\psi_e = \psi_e^+ + \psi_e^-$ and, eventually

$$\psi_e^+ = \frac{1}{2}K(\langle tr(\boldsymbol{\varepsilon})\rangle^+)^2 + \mu(\boldsymbol{\varepsilon}_e^D : \boldsymbol{\varepsilon}_e^D) \text{ and } \psi_e^- = \frac{1}{2}K(\langle tr(\boldsymbol{\varepsilon})\rangle^-)^2 \tag{7}$$

Using Eq. (6), the Ginzburg-Landau evolution equation of the phase-field s becomes:

$$\frac{1}{M}\dot{s} = -2s\langle\psi_e^+ + \psi_p - \psi_c\rangle^+ - 2\frac{\psi_c}{\zeta}\left[\ s - 1 - 4\kappa^2\Delta s\right] \tag{8}$$

3 Numerical Example

The numerical implementation of the phase-field model for fracture is applied to the crack propagation in the brittle-ductile multilayered composite. A square plate of length 10 mm, containing a horizontal notch of length 0.5 mm, is subjected to uniaxial tension loading by prescribing a vertical displacement on the top and bottom edge boundaries (Fig. 1). The material parameters are chosen as E=205000 MPa, v=0.3, σ_y=245 MPa, H=760 MPa, κ=0.1 mm, η=0.0001, G_c=7 MPa.mm, ζ=1 and M=10^6 1/MPa.s. In this simulation, the influence of the ductile materials on the response of the multilayered composite is investigated. Therefore, the influence of the critical fracture work density ψ_c is analysed since it is the controller of the ductile failure process. To this aim, four different values for the critical fracture work density ψ_c are used. The obtained force displacement responses are depicted in Fig. 2. It is obvious that the load bearing capacity has become greater by increasing the critical fracture work density ψ_c. On the other hand, the level of the maximal force does not change as the crack firstly initiates and propagates in the brittle material. It can be observed that the fracture strength can be improved by the increasing ψ_c.

The Fig. 3 shows that the crack propagates straightly horizontal in the brittle material, then the crack follows the path with the angle of 45 degrees in the ductile material. It can also be observed that the crack path is nearly unaffected by the value of the critical fracture work density ψ_c.

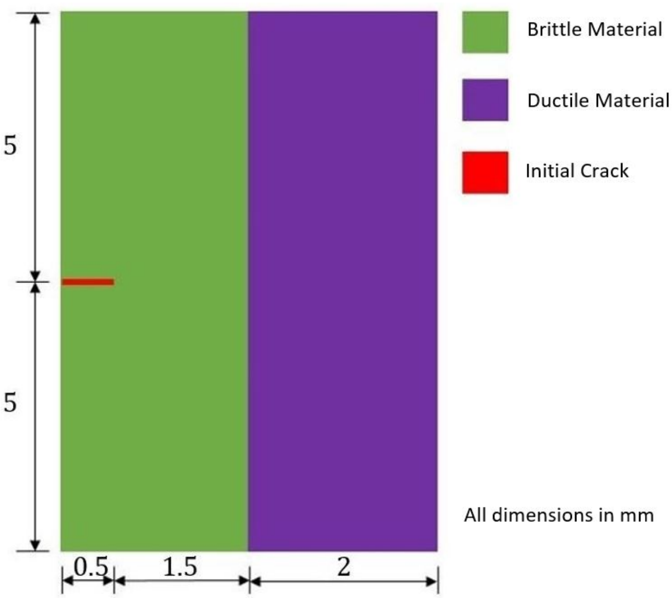

Figure 1. The notched elasto-plastic multilayered composite: the geometry and associated dimensions

Conclusions

The aim of this paper was to analyse the fracture-mechanics performance of the brittle-ductile multilayered composite using the phase-field method. Therefore, the utilize phase-field model of brittle fracture and ductile fracture are described in detail. The achieved results from this simulation of both the crack paths and force displacement responses from the phase-field models proved that the ductile material has good ability to improve the mechanical performance of the multilayered composite since the fracture strength can be improved by increasing the critical fracture work density ψ_c. In other words, the results suggest that the ductile layer can absorb more fracture energy. Finally, it is noteworthy that the fracture-mechanics performances of the proposed multilayered composite should be compared with experimental data which need to be investigated further. Moreover, the damage and failure analysis of heterogeneous materials

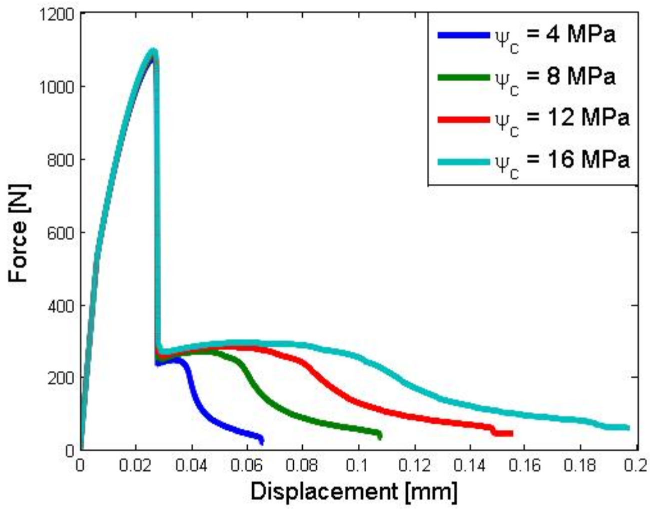

Figure 2. Force-displace curves for the different value of the critical fracture work density ψ_c

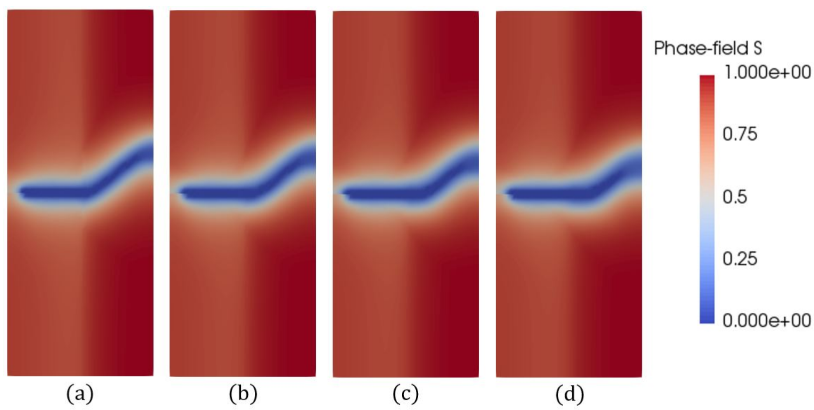

Figure 3. The final damage phase field distribution in the notched elasto-plastic multilayered composite using (a)$\psi_c = 4$ MPa , (b)$\psi_c = 8$ MPa, (c)$\psi_c = 12$ MPa and (d)$\psi_c = 16$ MPa

using phase-field modelling should be investigated further.

Acknowledgements

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