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A Partitioned Approach for Fluid-Structure Interaction Using NURBS-Enhanced Finite Elements and Isogeometric Analysis

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Micro Abstract
In the presented work the Deforming Spatial Domain/Stabilized Space-Time method extended with NEFEM is coupled with a semi-discrete IGA solver. IGA exploits the geometric properties of NURBS for numerical analysis. However, parametrizing the 3D domains typically needed for fluid dynamics is still challenging. Instead, NEFEM suggests the use of standard finite elements in the interior domain and special elements along the boundaries. A simplified data transfer and an error reduction are achieved.

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Introduction

Engineering design via CAD software relies on Non-Uniform Rational B-Splines (NURBS) as a means for representing and communicating geometry. Therefore, in general, a NURBS representation of a given geometry can be considered the exact description. The development of isogeometric methods has made this exact geometry available to analysis methods [3]. Isogeometric analysis has been particularly successful in structural analysis; one reason being the widespread use of two-dimensional finite elements in this field. For fluid dynamics, where three-dimensional analysis is usually indispensable, isogeometric methods are more complicated, yet not impossible, to apply in a general fashion. We propose a method that enables the solution of fluid-structure-interaction with a spline description of the interface [2]. On the structural side, the spline is used in an isogeometric setting. On the fluid side, the same spline is used in the framework of a NURBS-enhanced finite element method (extension of [5]). The coupling of the structural and the fluid solution is greatly facilitated by the common spline interface. The use of the identical spline representation for both sides permits a direct transfer of the necessary quantities, all the while still allowing an adjusted, individual refinement level for both sides.

1 Numerical Methods

Within the proposed partitioned FSI solver, NURBS-based isogeometric analysis is applied on the structural side. On the fluid side, we apply the Deforming Spatial Domain/Stabilized Space-Time method [7] with NURBS enhanced Finite Elements, in which the original NEFEM formulation was slightly modified.

1.1 The Modified Version of NEFEM Used in the FSI Context

The NEFEM implementation utilized in this work is based on the same idea as it was proposed by [4]. The differences lie in the fact that (1), we employ the space-time version, as derived in [6], and (2), we refrain from using Cartesian FEM, but compute both the shape functions and the position of the integration points on a — now triangular — reference element. What remains unchanged is the overall concept of adjusting of the position of the integration points to the curved NURBS shape as well as the representation of the unknown solution with Lagrange polynomials — even if restricted to linear polynomials in our case.

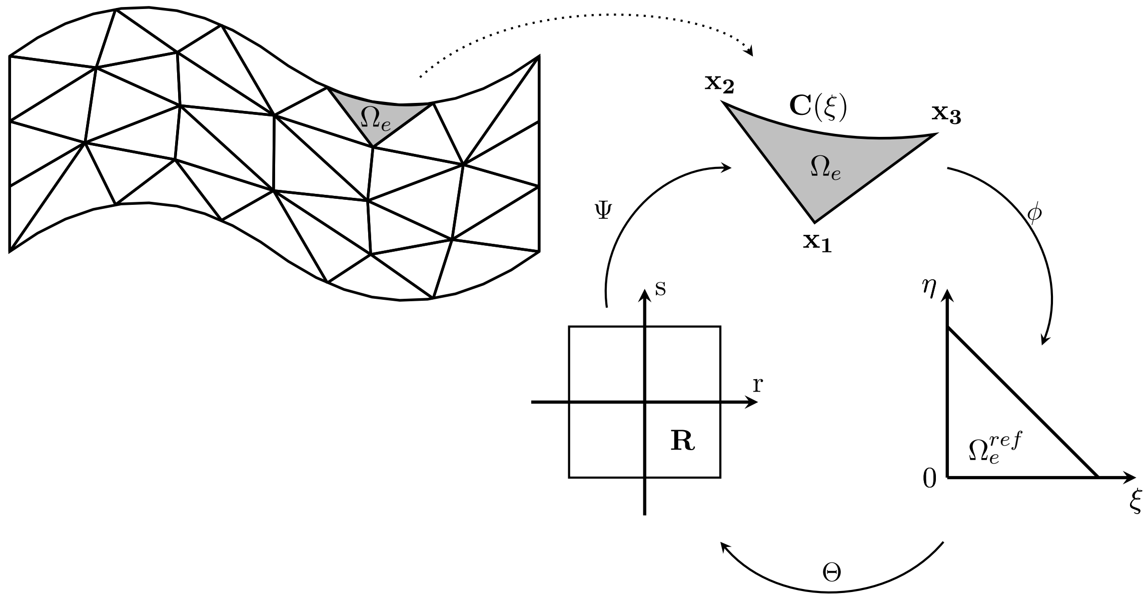

Figure 1. The Triangle-Rectangle-Triangle mapping from the reference triangle to the global triangle with one curved edge was derived from the original mapping from the reference bi-unit square to the curved triangle. By incorporating the bi-unit square, it can still be ensured that the NURBS direction and the interior direction are clearly separated, thus leading to straight interior edges even though the boundary edge is curved. $\boldsymbol{\Psi}$ is the mapping utilized in [4], $\boldsymbol{\Phi}$ is the new TRT mapping.

The derivation of the mapping is illustrated in Figure 1. It is used for both the definition of the shape function and the placement of the quadrature points. Figure 2 compares the p-FEM mapping from [4] with the modified mapping. In principle, the two mappings lead to a similar — although still different — distribution of quadrature points. Note however, that the p-FEM mapping has a singularity at one of the boundary nodes; thus excluding this point in case of a boundary integral. The TRT mapping has its singularity at the interior node.

2 Partitioned Solution Approach

2.1 Coupling Conditions at the Fluid-Structure Interface

The governing equations of fluid and structure need to be connected in order to represent the interaction between the two components. For a consistent coupling, the following physical requirements are essential: (1) geometric compatibility between the fields, (2) kinematic and dynamic conditions at the shared interface Γ_{FS}, to ensure conservation of mass, momentum and energy. This leads to the following coupling conditions on Γ_{FS}:

Kinematic continuity:

$$\begin{aligned} \mathbf{d}^f(\mathbf{x},t) &= \mathbf{d}^s(\mathbf{x},t) \qquad \text{on} \quad \Gamma_{FS}, \\ \mathbf{u}^f(\mathbf{x},t) &= \dot{\mathbf{d}}^s(\mathbf{x},t) \qquad \text{on} \quad \Gamma_{FS}. \end{aligned} \tag{1}$$

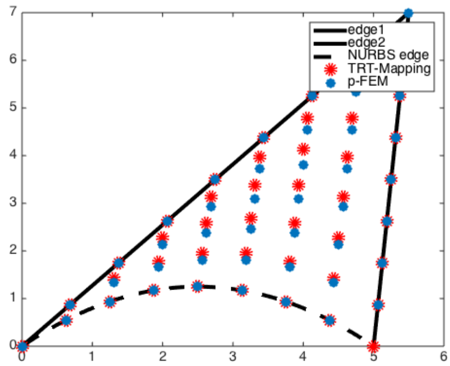

Figure 2. Comparison of the quadrature point placement for the p-FEM and the TRT mapping for an element with one circular edge

These coupling conditions ensure the continuity of displacements and velocities across the interface.

Dynamic continuity:

$$\boldsymbol{\sigma}^f(\mathbf{x}, t) \cdot \mathbf{n}^f = -\boldsymbol{\sigma}^s(\mathbf{x}, t) \cdot \mathbf{n}^s \qquad \text{on} \quad \Gamma_{FS}. \tag{2}$$

In agreement with Newton's third law – *Actio and Reactio* – this coupling condition enforces continuity of fluid ($\boldsymbol{\sigma}^f$) and structural stresses ($\boldsymbol{\sigma}^s$) at the interface Γ_{FS}.

2.2 Temporal Coupling

For the temporal coupling, we employ a strong coupling approach. The coupling conditions are fulfilled via fixed-point iterations between the structure and fluid within one time step until convergence is obtained. This information enters the respective other simulation as a boundary condition. In the sense of a Neumann-Dirichlet approach, stresses are transfered from structure to fluid and deformations from fluid to structure.

2.3 Spatial Coupling via Direct Integration

If NEFEM is applied on the fluid side and IGA on the structural side, the exact geometry of the coupling interface is available on both sides. Thus, a direct integration can be applied to obtain the fluid forces in the sense of a weighted residual method [1].

1) Calculation of discrete forces: Using the NURBS basis functions R_i as test functions, the fluid forces $\mathbf{F}_i^s$ on the right-hand-side of the structural problem can be formulated as follows:

$$\mathbf{F}_i^s = \int_{\Gamma_{FS}} R_i(\Theta) \left(\boldsymbol{\sigma}^f(\Theta) \cdot \mathbf{n}(\Theta) \right) \, d\Theta. \tag{3}$$

Following this idea, the forces are already available on every control point and can be directly used as a right-hand-side for the system of equations of the structural problem. An additional projection method is not needed anymore.

2) Transfer of deformation onto the fluid: The computed structural deformation has to be transferred back to the fluid in order to account for the mesh deformation. Once again, this procedure is straightforward due to the use of IGA. The deformation is computed at every spline control point. Knowing the parametric coordinates, we have to sum up the contributions of all control points that have non-zero NURBS basis function at that point. This is done in the following way:

$$\mathbf{d}_i^f = \sum_j^{n_{CP}} R_j\left(\Theta_i\right) \mathbf{d}_j^s. \tag{4}$$

Here, $\mathbf{d}_j^s$ is the computed displacement on the structural side — a displacement applied to the control points —, whereas $\mathbf{d}_i^f$ is the displacement of fluid node i — a displacement applied on FE node level.

Conclusions

A novel coupling scheme for FSI problems for spline-based methods is presented. Building on the recent advances in structural mechanics based on isogeometric analysis, the idea is to involve the splines also in the flow simulations. This was achieved by employing the NURBS-Enhanced Finite Element Method for the fluid.

A new geometrical mapping, ensuring that Dirichlet boundary conditions are fulfilled on the spline-based surface, was presented. With that, a direct transfer of the necessary coupling variables is possible, as the interface description is identical for both the structure and the fluid. In addition to the simplified implementation of the coupling, an increased accuracy of the flow solution — and with this the FSI solution — was to be expected due to the exact interface representation.

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