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A Novel Parameter Identification Toolbox for the Selection of Hyperelastic Constitutive Models from Experimental Data

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Micro Abstract
This paper presents a novel parameter identification toolbox based on various multi-objective optimization strategies for the selection of the best constitutive models from a given set of homogeneous experiments. The toolbox aims at providing an objective model selection procedure along with the material parameters for the rubber compound at hand. To this end, we utilize the multi-objective optimization using genetic algorithm of MATLAB. For the validation purposes, we use 10 constitutive laws.

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Introduction

The micromechanically based network models are known to be superior over the purely phenomenological models in the analysis of unfilled rubber. However, various factors play a crucial role in the determination of the appropriate constitutive model for the analysis of technical rubbers. These are; (i) the number of available experiments under various loading conditions, (ii) maximum stretch level expected in the critical loading scenarios of the rubber component, and (iii) percentage of fillers and additives that might distort the mechanical response from that of an ideal rubber. Moreover, the number of material parameters to be identified should be as low as possible and the parameters should be physically interpretable. Although the recent reviews have been very useful for the assessment of the strengths and weaknesses of various constitutive models, especially on the modeling of uncrosslinked ideal rubber, it is still a challenging endeavour for engineers to decide the best constitutive model for the specific rubber compound at hand [6, 13]. This paper presents a novel parameter identification toolbox based on various multi-objective optimization strategies for the selection of the best constitutive models from a given set of uniaxial tension, pure shear and equibiaxial tension experiments [14]. The toolbox aims at providing an objective model selection procedure along with the material parameters for the rubber compound at hand. To this end, we utilize the multi-objective optimization using genetic algorithm. For the validation purposes, we select the best ten models sorted according to the quality of fit expression provided in the manuscript, see also [6, 9]. The multi-objective optimization toolbox is shown to be a very efficient tool for the parameter identification and objective constitutive model selection procedure. We employed the developed optimization procedure for the determination of best hyperelastic constitutive models with respect to the quality of fit to Treloar's data [14].

1 Optimization Procedure

The complexity of the parameter identification for each model is unique since they have different number of parameters and different notion or paradigm of modeling (i.e. mathematical, phenomenological or physical modeling). A model can only provide a limited portrait of physical phenomena since consideration all the way down to the atomic scale is not possible by the current technological means. Therefore, lower or upper bounds of the search domain cannot be determined or deduced from the model itself. Hence, every optimization procedure has to be bound with sufficiently large search domain but not infinity if we are to try to find the global minimum. However, certain parameters due to modeling procedure has natural bounds. Genetic algorithm provides a straightforward mechanism to obtain the best parameters minimizing the cost function given the search domain. In this work, we developed a genetic algorithm that is adaptable to models with different number of parameters and different search domains. After finding the parameters via the genetic algorithm we provided these values to MATLAB Optimization Toolbox nonlinear programming tool as initial parameters. Concurrently, the MATLAB Optimization Toolbox is utilized for the identification of the best parameter set. Both results are compared and the parameter set with the best quality of fit measure is selected. This ensures that the best fit is the absolute minimum in the search domain. We observed quite similar parameters that reported in review papers [6, 9]. The error expressions for uniaxial tension (UT), equibiaxial tension (ET), and pure shear (PS) loading modes are provided as follows,

$$e_{ut} = \frac{1}{m_{ut}} \sum_{i=1}^{m_{ut}} \left(P_{11}^{exp} - P_{11}\right)^2, \quad e_{et} = \frac{1}{m_{et}} \sum_{i=1}^{m_{et}} \left(P_{11}^{exp} - P_{11}\right)^2, \quad e_{ps} = \frac{1}{m_{ps}} \sum_{i=1}^{m_{ps}} \left(P_{11}^{exp} - P_{11}\right)^2. \quad (1)$$

As opposed to mainstream single-objective optimization procedure, in this work we used a multi-objective optimization. We weight the uniaxial, equibiaxial, and pure shear error expressions with three weight parameters (w_1, w_2, w_3), resulting in the cost function

$$E = w_1 e_{ut} + w_2 e_{et} + w_3 e_{ps} \,, \quad (2)$$

where $w_1 + w_2 + w_3 = 1$. Note that, the optimization procedure takes the weight factors w_1 and w_2 as optimization parameters in addition to the material parameters. This fact is the novel aspect of the procedure. Hence, we signify the different sensitivity of each constitutive model to different deformation modes in error expression. This way, inherent deformation mode dependencies of constitutive models are buried deep within the optimization procedure.

1.1 Results and Conclusion

In this study, 10 best hyperelastic material models are chosen among 38 hyperelastic constitutive models sorted according to the best quality of fit measure. In order to make an unbiased comparision between the considered hyperelastic material models, a fit of quality measure with the following form is proposed

$$\chi^2 = \sum_{i=1}^{m} \frac{\left(P_{11}^{exp} - P_{11}\right)^2}{P_{11}^{exp}} \,, \quad (3)$$

where m is the total number of experimental data points. χ^2 measure is considered for each model in three regions of deformation. Low stretch $(1 < \lambda \leqslant \frac{1}{3}\lambda_{max})$, moderate stretch $(1 < \lambda \leqslant \frac{2}{3}\lambda_{max})$, and large stretch $(1 < \lambda \leqslant \lambda_{max})$ regions are denoted as region 1, region 2, and region 3, respectively. Here, λ_{max} is the maximum stretch value attained during the experiment under uniaxial tension, equibiaxial tension and pure shear loading, respectively. The obtained results are summed over loading cases for each region to yield total quality of fit measure for each region. Therefore, the total quality of fit for the third region would be a reasonable measure for comparing models in an ubiased way. Obtained results are listed in Tables 1-2 according to the goodness of fit for full range of extension with simultaneous fitting of uniaxial, equibiaxial, and pure shear data. Since the models have different quatify of fit values for different regions and region 3 encompasses all other regions, the comparison is done based on the quality of fit value in region 3. Related plots for fitting the material parameters with three sets of experimental data and uniaxial data (only) are presented in Figures 1-2, respectively. Uniaxial data fitting is carried out to show how well a model can predict other deformation modes if a single set of experiments are used for fitting.

Micro-sphere model [10]

$$\psi(\lambda) = \mu\left(\lambda_r \mathcal{L}^{-1}(\lambda_r) + \ln\frac{\mathcal{L}^{-1}(\lambda_r)}{\sinh\mathcal{L}^{-1}(\lambda_r)}\right) \text{ with } \lambda = \langle\bar{\lambda}\rangle_p := \left[\frac{1}{|\mathscr{S}|}\int_{\mathscr{S}} \bar{\lambda}^p \, dS\right]^{1/p}$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$\mu = 0.2939$ $N = 21.99$ $p = 1.462$ $U = 1.433$ $q = 0.0554$	$\mu = 0.2047$ $N = 31.33$ $p = 3.347$ $U = 1.315$ $q = 0.09197$

			Quality of fit				**Quality of fit**		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.2009	0.0032	0.0058	0.0086	0.0265	0.0043	0.0304	0.0520	0.0687
ET	0.5990	0.0006	0.0215	0.0217	0.0261	0.1010	0.0100	0.0277	0.8579
PS	0.2001	0.0003	0.0113	0.0115	0.0131	0.0013	0.0082	0.0196	0.0280
Total	1.0000	0.0041	0.0386	0.0418	0.0657	0.1066	0.0486	0.0993	0.9545

Alexander's model [1]

$$\psi = C_1 \int \exp[k(I_1-3)^2] dI_1 + C_2 \ln\left[\frac{(I_2-3)+\gamma}{\gamma}\right] + C_3(I_2-3)$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$C_1 = 0.1403$ $C_2 = 0.2588$ $C_3 = 0.002143$ $\gamma = 5.991$ $k = 0.0003464$	$C_1 = 0.1314$ $C_2 = -0.3305$ $C_3 = 0.0985$ $\gamma = 5.814$ $k = 0.0003619$

			Quality of fit				**Quality of fit**		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.3470	0.1073	0.0073	0.0096	0.0309	0.1053	0.0149	0.0185	0.0391
ET	0.3495	0.0079	0.0231	0.0242	0.0266	828.50	0.0345	7.7825	409.96
PS	0.3036	0.0036	0.0122	0.0126	0.0140	1.4855	0.0209	0.0487	1.0460
Total	1.0000	0.1188	0.0427	0.0465	0.0715	830.10	0.0703	7.8497	411.05

Extended-tube model [7]

$$\psi = \frac{G_c}{2}\left[\frac{(1-\delta^2)(D-3)}{1-\delta^2(D-3)} + \ln\left(1-\delta^2(D-3)\right)\right] + \frac{2G_e}{\beta^2}\sum_{A=1}^{3}\left(\lambda_A^{-\beta}-1\right) \quad \text{with} \quad D = \sum_{A=1}^{3}\lambda_A^2$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$G_c = 0.1956$ $\delta = 0.09532$ $G_e = 0.1949$ $\beta = 0.1759$	$G_c = 0.1105$ $\delta = 0.1039$ $G_e = 0.2047$ $\beta = -1.779$

			Quality of fit				**Quality of fit**		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.2002	0.1417	0.0027	0.0055	0.0325	0.0833	0.0281	0.0311	0.0466
ET	0.5997	0.0235	0.0163	0.0352	0.0392	0.6622	0.0731	0.2349	0.5210
PS	0.2000	0.0023	0.0083	0.0084	0.0096	0.0303	0.0455	0.0692	0.0754
Total	1.0000	0.1675	0.0274	0.0491	0.0812	0.7758	0.1467	0.3352	0.6430

Shariff's model [12]

$$\psi = \sum_{i=1}^{n} \alpha_i \phi_i$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$E = 1.124$ $\alpha_1 = 0.9174$ $\alpha_2 = 0.03712$ $\alpha_3 = 7.872e-05$ $\alpha_4 = 0.02361$	$E = 1.254$ $\alpha_1 = -4.12$ $\alpha_2 = 0.05454$ $\alpha_3 = 10.6$ $\alpha_4 = -0.3131$

			Quality of fit				**Quality of fit**		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.2065	0.1611	0.0117	0.0178	0.0470	0.0952	0.0048	0.0087	0.0276
ET	0.3182	0.0102	0.0243	0.0286	0.0305	2.9e10	1.3098	5.4e06	1.3e10
PS	0.4753	0.0055	0.0111	0.0119	0.0143	4.4e05	0.5884	807.88	2.8e05
Total	1.0000	0.1768	0.0471	0.0583	0.0918	2.9e10	1.9029	5.4e06	1.3e10

Carroll's model [3]

$$\psi = AI_1 + BI_1^4 + C\sqrt{I_2}$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$A = 0.1458$ $B = 3.191e-07$ $C = 0.1044$	$A = 0.1574$ $B = 2.916e-07$ $C = 0.1005$

			Quality of fit				**Quality of fit**		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.3333	0.1668	0.0119	0.0156	0.0475	0.3021	0.0177	0.0582	0.1038
ET	0.3333	0.0127	0.0359	0.0465	0.0476	0.0301	0.0248	0.0333	0.0455
PS	0.3333	0.0065	0.0199	0.0221	0.0235	0.0423	0.0116	0.0153	0.0453
Total	1.0000	0.1860	0.0678	0.0842	0.1186	0.3745	0.0540	0.1068	0.1946

Table 1. Obtained parameters, errors, and quality of fit values for the first five models.

Chevalier and Marco's model [4]

$$\frac{\partial\psi}{\partial I_1} = \sum_{i=0}^{n} a_i\left(I_1 - 3\right)^i \quad and \quad \frac{\partial\psi}{\partial I_2} = \sum_{i=0}^{n} \frac{b_i}{I_2^i}$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$a_0 = 0.1585$ $a_1 = -0.002321$ $a_2 = 0.0001175$ $b_0 = 0.002048$ $b_1 = 0.2266$ $b_2 = -0.4902$	$a_0 = 0.1833$ $a_1 = -0.004043$ $a_2 = 0.0001431$ $b_0 = 0.0003649$ $b_1 = 0.001588$ $b_2 = -0.3825$

			Quality of fit				Quality of fit		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.3226	0.3114	0.0161	0.0255	0.0834	0.2483	0.0387	0.0552	0.1030
ET	0.5643	0.0236	0.0290	0.0324	0.0428	0.8830	0.1207	0.3281	0.7348
PS	0.1131	0.0108	0.0138	0.0174	0.0221	0.0876	0.0681	0.0860	0.1267
Total	1.0000	0.3458	0.0588	0.0753	0.1482	1.2190	0.2275	0.4693	0.9645

Ogden's model [11]

$$\psi = \sum_{i=1}^{3} \frac{\mu_n}{\alpha_n}\left(\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3\right)$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$\alpha_1 = 1.873$ $\alpha_2 = 7.991$ $\alpha_3 = -1.845$ $\mu_1 = 0.364$ $\mu_2 = 2.706e-06$ $\mu_3 = -0.01659$	$\alpha_1 = 1.873$ $\alpha_2 = 7.991$ $\alpha_3 = -1.845$ $\mu_1 = 0.364$ $\mu_2 = 2.701e-06$ $\mu_3 = -0.0166$

			Quality of fit				Quality of fit		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.3333	0.1720	0.0161	0.0320	0.0631	0.1723	0.0161	0.0320	0.0630
ET	0.3333	0.0423	0.0371	0.0593	0.0736	0.0423	0.0370	0.0592	0.0737
PS	0.3333	0.0106	0.0177	0.0190	0.0237	0.0107	0.0177	0.0190	0.0237
Total	1.0000	0.2249	0.0709	0.1103	0.1604	0.2253	0.0709	0.1102	0.1604

Haines-Wilson's model [5]

$$\psi = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + C_{11}(I_1 - 3)(I_2 - 3) + C_{02}(I_2 - 3)^2 + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$C_{10} = 0.1845$ $C_{01} = 0.007551$ $C_{11} = -0.0001221$ $C_{02} = 1.989e-06$ $C_{20} = -0.001895$ $C_{30} = 4.598e-05$	$C_{10} = 0.000145$ $C_{01} = 0.2141$ $C_{11} = 0.03559$ $C_{02} = 8.547e-05$ $C_{20} = -0.008685$ $C_{30} = 6.944e-05$

			Quality of fit				Quality of fit		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.6420	0.2868	0.0430	0.0827	0.1284	0.1856	0.0054	0.0106	0.0478
ET	0.2545	0.0079	0.0228	0.0297	0.0303	3.7e05	0.0629	550.55	1.7e05
PS	0.1036	0.0255	0.0088	0.0154	0.0289	461.76	0.0212	2.2490	294.44
Total	1.0000	0.3202	0.0747	0.1278	0.1876	3.7e05	0.0895	552.81	1.7e05

Biderman's model [2]

$$\psi = C_{10}(I_1 - 3) + C_{01}(I_2 - 3) + C_{20}(I_1 - 3)^2 + C_{30}(I_1 - 3)^3$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$C10 = 0.1832$ $C01 = 0.002751$ $C20 = -0.001717$ $C30 = 4.351e-05$	$C10 = 0.1726$ $C01 = 0.003676$ $C20 = -0.00175$ $C30 = 4.542e-05$

			Quality of fit				Quality of fit		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.2007	0.3167	0.0353	0.0738	0.1252	0.2546	0.0171	0.0312	0.0806
ET	0.5991	0.0629	0.0299	0.0644	0.0864	0.1399	0.0418	0.1079	0.1533
PS	0.2002	0.0118	0.0118	0.0142	0.0203	0.0584	0.0195	0.0213	0.0559
Total	1.0000	0.3914	0.0770	0.1525	0.2319	0.4529	0.0783	0.1603	0.2898

Lambert-Diani and Rey's model [8]

$$\psi = \int \exp\left\{\sum_{i=0}^{3} a_i(I_1 - 3)^i\right\} dI_1 + \int \exp\left\{\sum_{i=0}^{2} b_i(\ln I_2)^i\right\} dI_2$$

	Simultaneous data fitting	Uniaxial data fitting
Parameters	$a_0 = 0.1644$ $a_1 = -0.003781$ $a_2 = 0.0003629$ $b_0 = 257.5$ $b_1 = -7.6$	$a_0 = 0.1529$ $a_1 = -0.003198$ $a_2 = 0.0003786$ $b_0 = 250.6$ $b_1 = -7.42$

			Quality of fit				Quality of fit		
	Weights	Error	Region 1	Region 2	Region 3	Error	Region 1	Region 2	Region 3
UT	0.1215	0.1781	0.0103	0.0446	0.0746	0.1010	0.0017	0.0036	0.0242
ET	0.6608	0.6373	0.0065	0.1164	0.4102	0.9442	0.0065	0.1897	0.6232
PS	0.2177	0.0050	0.0058	0.0085	0.0105	0.0341	0.0056	0.0218	0.0382
Total	1.0000	0.8204	0.0226	0.1695	0.4953	1.0793	0.0138	0.2151	0.6856

Table 2. Obtained parameters, errors, and quality of fit values for the last five models.

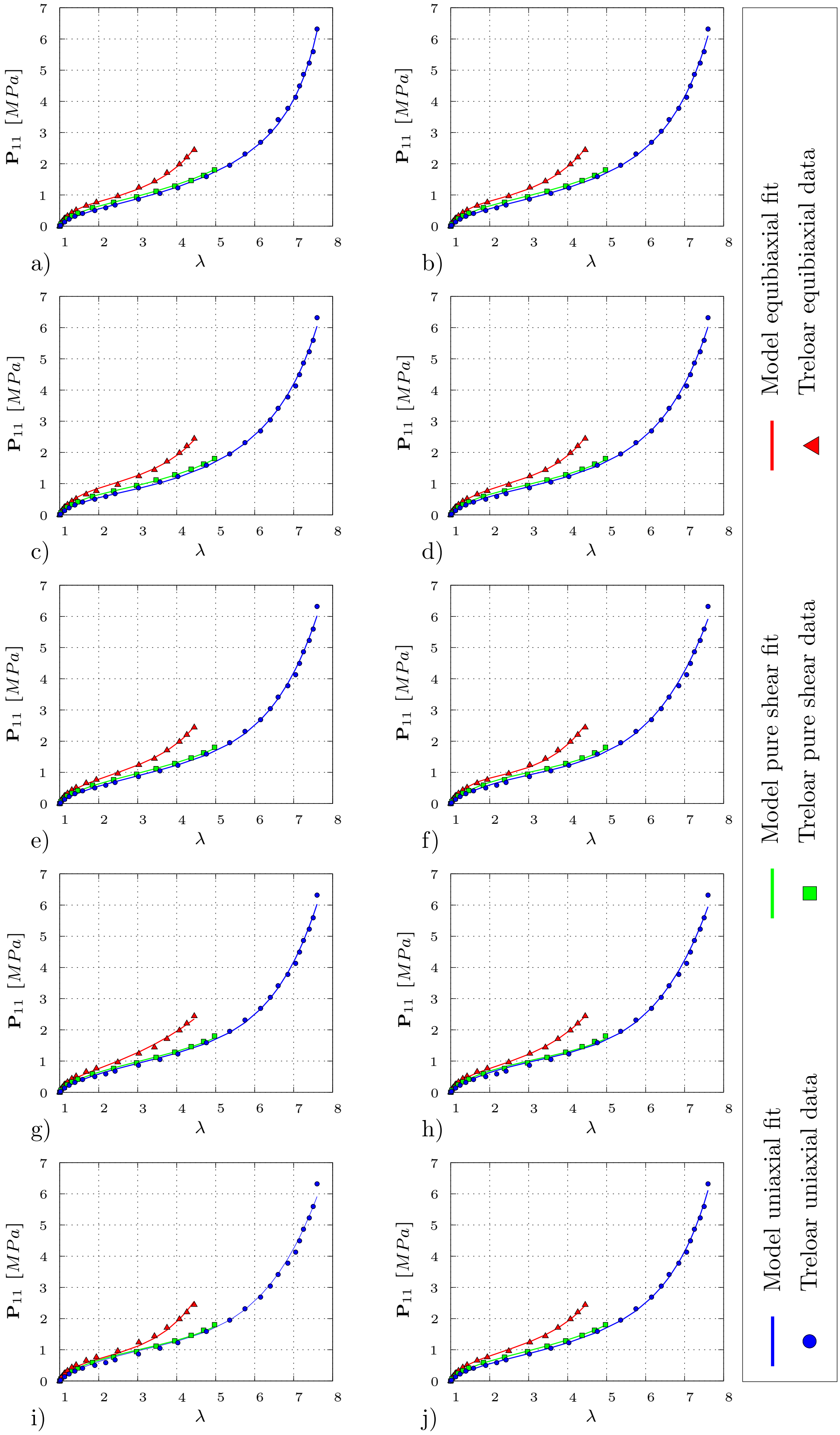

Figure 1. Results of simultaneous fitting of uniaxial, equibiaxial, and pure shear (Treloar) data to 10 models: a) Micro-sphere model, b) Alexander's model, c) Extended-tube model, d) Shariff's model, e) Carroll's model, f) Chevalier and Marco's model, g) Ogden's model, h) Haines-Wilson's model, i) Biderman's model, j) Lambert-Diani and Rey's model.

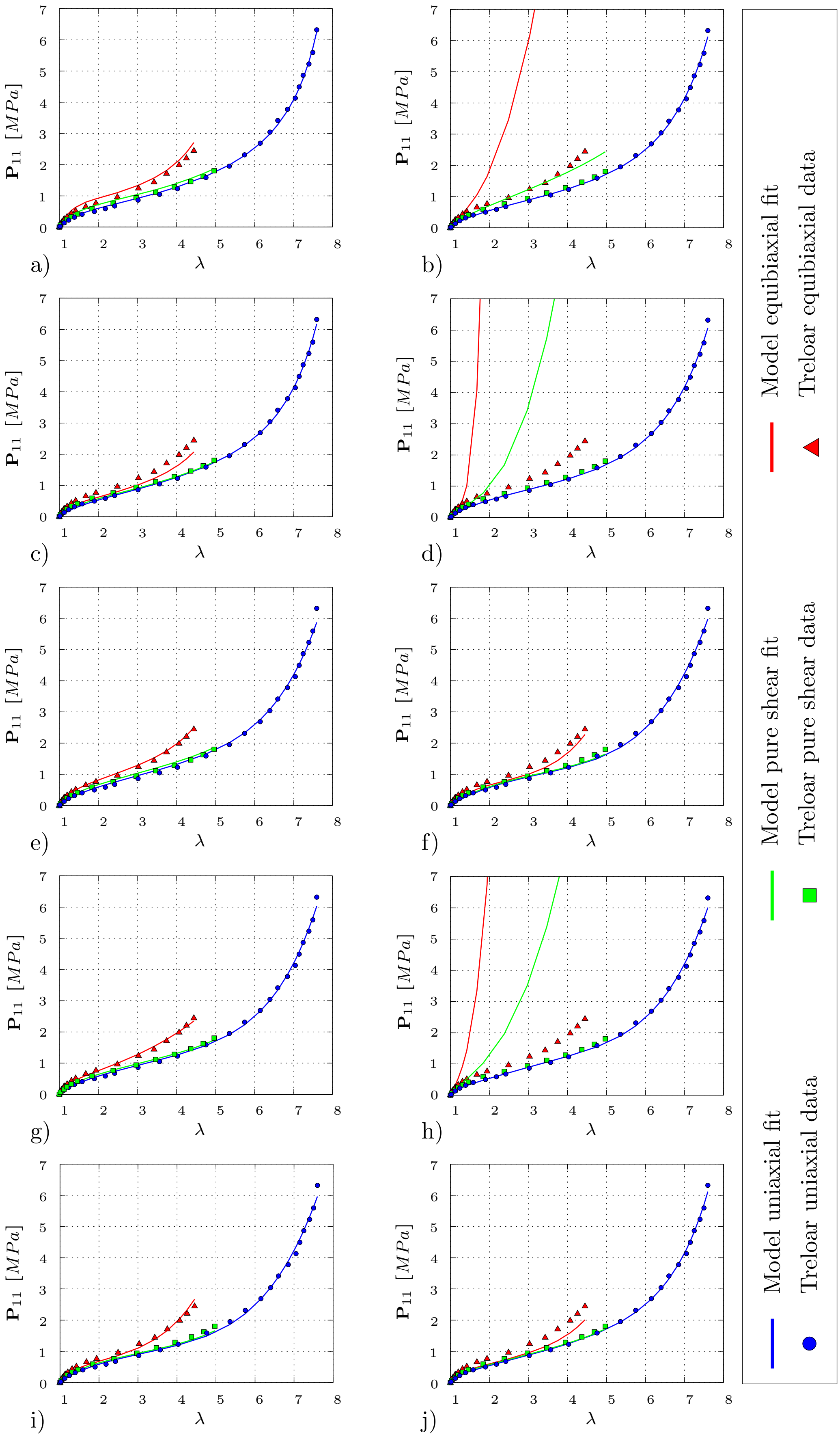

Figure 2. Results of fitting of uniaxial (Treloar) data to 10 models: a) Micro-sphere model, b) Alexander's model, c) Extended-tube model, d) Shariff's model, e) Carroll's model, f) Chevalier and Marco's model, g) Ogden's model, h) Haines-Wilson's model, i) Biderman's model, j) Lambert-Diani and Rey's model.

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