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Optimization of topology and shape, combining phase field modelling and discrete stochastic algorithms

Alexander Keller[1]*, Ingo Münch[1] and Werner Wagner[1]

Micro Abstract
For the design of frame structures in civil engineering we are interested in an approach to combine topology and shape optimization. We use a phase field model to generate topology as design concept first. However, it is not possible to estimate the overall fitness of obtained topologies concerning more complex criteria required in civil engineering. Therefore, as a second step, shape optimization with metaheuristic methods considering the normative constraints is performed.

[1]Institute for Structural Analysis, Karlsruhe Institute of Technology, Karlsruhe, Germany
***Corresponding author**: alexander.keller@kit.edu

Introduction

As the slogan 'form follows function' states, the design of a load bearing structure should be based on its purpose, which is to balance loadings. An optimized design is ruled by an efficient flow of forces and complies with all necessary standards. However, most real-world problems of optimization cannot be solved analytically. Thus, numerical methods are of great practical importance. The first step in designing a structure is to define a topology. Since the flow of forces can be found within elasticity theory, it is not essential to account for fracture or plasticity first. However, stress limitations should be considered in a second step, when the topology is found and simplified, if desired. With this, the shape of this topology is optimized with another objective function, where normative side conditions are taken into account.

1 Principle of the two step optimization

In our opinion, the most neutral conceptual design for load bearing structures is a homogeneously filled region $\mathcal{B}$ of material. Then, on the basis of phase field modeling (PFM), we reduce the filling degree of material by evolution of phases. In the second step, the final topology of the PFM is used to set up a simplified model with single nodes and beams. The subsequent shape optimization considers normative constraints like yield stress to determine the discrete values of optimization parameters, e.g. the cross section of each member. Since the optimization of framework structures with a fixed set of optimization parameters belongs to the combinatorial problems, metaheuristic optimization methods as the Evolutionary Algorithm (EA) are used.

1.1 Topology optimization with the phase field model

Many numerical methods for topology optimization have been developed, see e.g., [3, 7], since Bendsøe and Kikuchi [1, 2] proposed the material distribution concept instead of discrete values for voids and material. Our approach additionally assumes that local failure is predictable by the equivalent stress σ_V from the von Mises stress criterion. Nevertheless, we avoid to consider a certain stress limit $\sigma_V \leq f_y$ as suggested in [4, 5]. Our algorithm homogenizes σ_V in the evolving structure by seeking the minimum of an energy function. The evolution is ruled by an

Allen-Cahn equation concerning φ, which is the phase field parameter coupling to the density ρ_φ and stiffness $\mathbb{C}_\varphi$ of the material:

$$f(\varphi) = \frac{e^{\alpha\varphi}}{e^{\alpha\varphi}+1}\ , \qquad \rho_\varphi = f_\varphi\, \rho_0\ , \qquad \mathbb{C}_\varphi = f(\varphi)\, \mathbb{C}_0. \tag{1}$$

Here ρ_0 and $\mathbb{C}_0$ is the density and the elasticity matrix of the employed material. The model evolves voids since the favorable states for φ are given by a well potential with minimal energy at $\varphi = -1$ and $\varphi = 1$. Regions with $\mathbb{C}_\varphi \to 0$ are denominated voids, whereas regions with $\mathbb{C}_\varphi \to \mathbb{C}_0$ are filled with material. The complexity of the evolving structure is controlled by two main parameters: the thickness of the diffuse interface zone between voids/material and the amount of external work on $\gamma\varphi$ during the process. It is comparable to external work for the injection or extraction of material. Since $\gamma(\sigma_V)$ drives the evolution process, it can be interpreted as "pressure" to adjust the material density in $\mathcal{B}$. We assume the total energy of the design domain $\mathcal{B}$ given by inner energy $\Psi(\varepsilon, \varphi, \mathrm{Grad}[\varphi])$, and external work, reading

$$\Pi(\mathbf{u}, \varphi, \mathrm{Grad}[\varphi]) = \int_{\mathcal{B}} \Psi(\varepsilon(\mathbf{u}), \varphi, \mathrm{Grad}[\varphi])\, \mathrm{dV} - \int_{\mathcal{B}} (\rho_\varphi\, \mathbf{b}\cdot\mathbf{u} + \gamma\,\varphi)\, \mathrm{dV} - \int_{\partial\mathcal{B}} (\mathbf{t}\cdot\mathbf{u} + y\,\varphi)\, \mathrm{dA}. \tag{2}$$

With $\mathbf{b}$ we denote external net forces, such that $\rho_\varphi\, \mathbf{b}$ is a body force, e.g., to account for the weight of material.

1.2 Shape Optimization with an Evolutionary Algorithm

This optimization method uses agents to scan the search area for feasible solutions and divides the shape optimization process into single steps called generations. The algorithm usually succeeds in finding acceptable solutions even for complex fitness landscapes and increased number of optimization parameters. The process starts from a population of randomly generated individuals, in which discrete values of the optimization parameters represent the genome of individuals. The fitness of individuals of each generation is evaluated. The objective function rating individuals is easy to extend and can account for arbitrary constraints.

Individuals with high fitness have increased probability to become the basis for new individuals of the next generation. The reproduction mechanisms of the EA are inspired by the mechanisms of evolution: cloning, mutation, recombination, and selection. Mutation is used to avoid convergence into local minima. Cloning preserves individuals with already good fitness. By recombining the genome of different individuals and filtering the resulting individuals with bad fitness per generation in the selection process, convergence to a solution is guaranteed. In our EA three parameter control the convergence rate of the algorithm: the number of individuals per generation, the reproduction rate and the ratio of mutation to recombination. These parameters influence computational time and quality of the optimization process.

We found that the simultaneous optimization of nodal coordinates and cross sections profits by calculation with a low number of individuals but a high reproduction ratio of 1 : 20 per generation. An adaptive mutation to recombination rate helps to scan the search area widely at the beginning of the optimization. However, it reduces its spread in the final stage, when good individuals have already been found.

2 Example with systematic investigation of topologies

In [6] we have discussed a single-span beam to explain the proposed workflow. Here we consider the two-span continuous beam of Fig. 1a, which is statically indetermined. The optimization of such systems is more challenging: reaction forces conduct the shape optimization, which itself yields the stiffness of members. However, the stiffness of members conduct the reaction forces in the system. Thus, the overall process is recursive.

Our computational model in Fig. 1b notes symmetry. As design space $\mathcal{B}$ we choose a rectangular

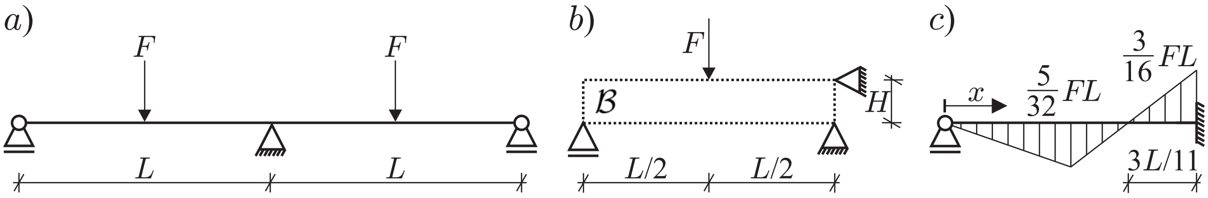

Figure 1. a) Full system with loading $F = 36.5\,kN$. b) Symmetric system and design space $\mathcal{B}$. c) Bending moment in case of $EI = const$.

with dimensions $L = 600\,cm$ and $H = 100\,cm$.
First, we consider the most simple topology, which is the single beam with $EI = const$. The associated bending moment is shown in Fig. 1c. Dimensioning with construction steel ($t = 1\,cm, f_y = 24\,kN/cm^2$) yields $19224\,cm^3$ of material to balance the load. Despite its simplicity, it still provides knowledge about the load carrying characteristics of solutions with more members. The bending moment changes its sign at $x_0 = 436.36\,cm$ and subsequently the required height of the cross section at this point reduces mathematically to zero. In other words, it is obvious that a hinge at x_0 not alter the bending moment in Fig. 1c. By introducing such a hinge, the system becomes statically determined such that the stiffness of the beam is without effect on the bending moment.

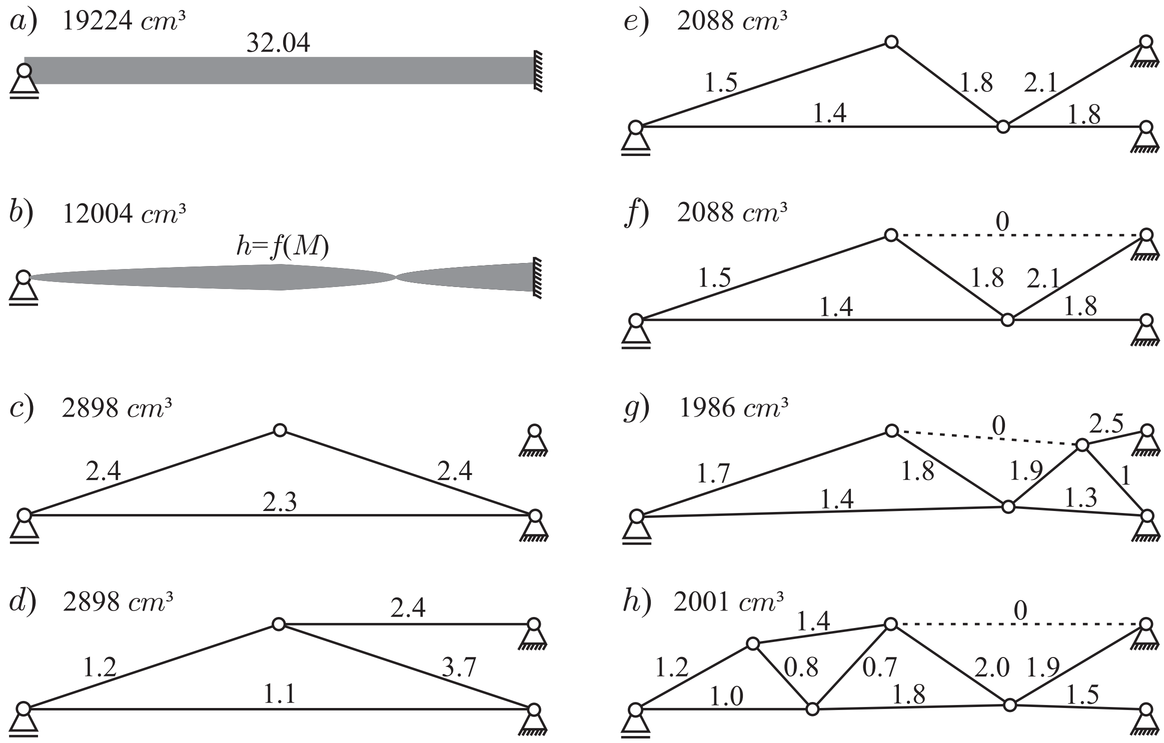

Figure 2. a) - h) Optimized topological variants with rising complexity and their total material consumption. Height of cross sections in $[cm]$.

This allows us to verify a two-beam topology, which can be an optimized beam, where the height is induced by the bending moment, see Fig. 2b. Therefore, the beam assimilates the bending moment distribution as shape. It requires $12004\,cm^3$ of material and the maximal height of the cross section is $32.04\,cm$ at the clamped support. However, it does not make use of the possible effective height given by the design space $\mathcal{B}$. The next level of topology is to evolve a truss structure making use of the effective height such that the bending moments are omitted. The cross sections of trusses are considered rectangular and each has a variable height h_i.
Topologies with rising number of members are shown in Fig. 2c) - h). It is possible for the EA to reduce the complexity of the truss structure by choosing the height $h = 0\,cm$ for members if

desired. The topology with the least material after shape optimization is shown in Fig. 2g. The EA reduces the number of trusses and shifts coordinates such that it reduces the effective height at x_0 similar to Fig. 2b.
The structure evolved by the phase field model, see Fig. 3d, yields a hybrid topology of Fig. 2e and Fig. 2g as solution. It also reduces the statical height at x_0. However, it does not coincide exactly with the investigated topologies of Fig 2 since it is a continuum model and does not reproduce hinges. By using the PFM, the topology in Fig. 2g is automatically found after shape optimization, such that for practical problems the efforts of comparing many different topologies can be omitted.

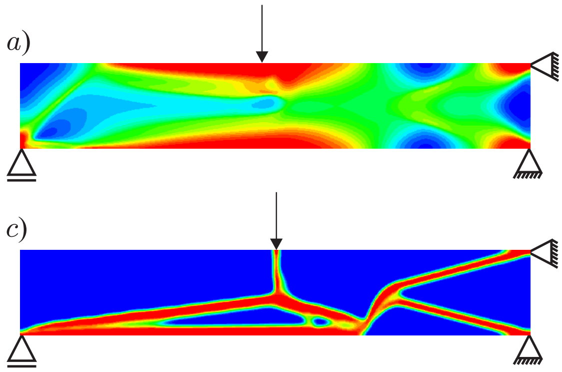

Figure 3. Evolution of the load bearing structure by PFM from a) - d).

3 Conclusions

We have compared different topologies with rising complexity of a statically indetermined system by material consumption. The example has shown, that the topology yielded by the PFM serves as a valuable basis for the subsequent shape optimization with EA such that the time to design an optimized structure can be reduced. An interface appears here as useful tool between both optimization steps to simplify the continuum model to a beam model with hinges, if desired. In the second step, stress limitations of a specified material and constraints are considered to dimension the evolved structure using relevant standards.

References

[1] M. Bendsøe. Optimal shape design as a material distribution problem. *Structural optimization*, 1(4):193–202, 1989.

[2] M. Bendsøe and N. Kikuchi. Generating optimal topologies in structural design using a homogenization method. *Comput. Methods Appl. Mech. Engrg.*, 71:197–224, 1988.

[3] K. Bletzinger and K. Maute. Towards generalized shape and topology optimization. *Eng. Opt.*, 29:201–216, 1997.

[4] M. Burger and R. Stainko. Phase-field relaxation of topology optimization with local stress constraints. *SIAM J. Control Optim.*, 45:1447–1466, 2006.

[5] P. Duysinx and M. Bendsøe. Topology optimization of continuum structures with local stress constraints. *Int. J. Numer. Meth. Engng.*, 43:1453–1478, 1998.

[6] A. Keller, I. Münch, and W. Wagner. Optimization of topology and shape by combining phase field modeling and discrete stochastic algorithms. *Submitted to Proceedings in Applied Mathematics and Mechanics.*

[7] A. Takezawa, S. Nishiwaki, and M. Kitamura. Shape and topology optimization based on the phase field method and sensitivity analysis. *Journal of Computational Physics*, 229:2697–2718, 2010.