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# Modeling three-dimensional anisotropic damage in organic sheet composites at large deformation

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**Micro Abstract**
Organic sheets consist of embedded interwoven rovings in a thermoplastic matrix. Loading results in a finite change of local reinforcement orientations with reversible and irreversible contributions. A constitutive model taking into account the large strain kinematics and damage evolution is presented. Mechanism-based damage formulations in both the reinforcements and the matrix are employed. Numerical examples demonstrate the features of the suggested material model.

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## Introduction

Many composites consist of a fabric structure embedded in a matrix material. In the present case organic sheets are considered. These are layered composites, where each layer consists of two sets of planar interwoven rovings (weft and warp), building a twill weave (cf. Figure 1), impregnated by a thermoplastic polypropylene matrix. The former define preferred directions in the material due to the reinforcement. These directions are known initially, e.g. by drape simulation or assumption. In general, the application of external mechanical loads results in a finite change of reinforcement orientation, containing reversible and irreversible contributions and hence yielding a differing stress-free state upon unloading. Hereby, the matrix is related to plastic deformation processes in the matrix-dominated regions of the composite material. The present work concentrates on the development of a computational model taking into account the aforementioned kinematic observations. Moreover, it refers to previous works enabling the incorporation of mechanism-based damage formulations in both the reinforcements and the thermoplastic matrix. The presentation closes with a numerical example demonstrating the features of the suggested material model.

## 1 Composite formulation

Materials with preferred directions perpendicular to each other are associated with orthotropic symmetry. In crystal plasticity it is often assumed that the initially known preferred directions (i.e. the crystal latices) do not change with the material directly (cf. Rice [7], Kröner and Teodosiu [1] and Mandel [2]). In the present case of weave reinforced composites, the preferred directions (in form of the rovings) change during loading, according to the material deformation (cf. Figure 1). In his work, Miehe [3] states that for such cases a finite plasticity formulation based on $\boldsymbol{G}^p = \boldsymbol{F}^{p^T} \boldsymbol{g} \boldsymbol{F}^p$ can be formulated. However, due to the occuring anisotropy, it is hardly possible to find a sound overall formulation for plasticity and damage onset in the material. To overcome this problem, the approach of the model presented here is to superpose two angulated preferred directions, coupled through an isotropic matrix. Hereby, the work of Reese [6] served as a guideline.

### 1.1 Material orientation and plastic intermediate configuration

In general, the initial structural tensors of a weave can be written as $\boldsymbol{M}_i = \boldsymbol{A}_i \otimes \boldsymbol{A}_i$ for each preferred direction $i \in [1, 2]$ (cf. Figure 1(a)). Given an arbitrary deformation expressed by the

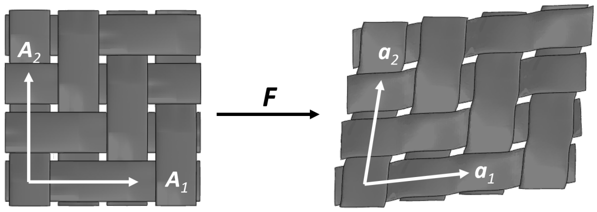


**Figure 1.** Change of material orientation due to deformation of weave-reinforced composite. The display of matrix domain is omitted. Initial material orientation $\boldsymbol{A}_i$ is mapped on deformed presentation $\boldsymbol{a}_i$ by the deformation gradient $\boldsymbol{F}$.

deformation gradient, $\boldsymbol{F} = \boldsymbol{F}^e \boldsymbol{F}^p$, the material orientation may change and will now be aligned with the deformed structural tensors (cf. Figure 1(b)), $\boldsymbol{m}_i = \boldsymbol{a}_i \otimes \boldsymbol{a}_i$, where $\boldsymbol{a}_i$ are the deformed preferred directions, defined by $\boldsymbol{a}_i = \boldsymbol{F}\boldsymbol{A}_i$. Regarding thermoplastic basic constituents, it can be expected to observe inelastic behavior during loading. In the following it assumed that this behavior is attributed to plastic effects in the matrix (index $m$), resulting in a plastic share of the deformation gradient, $\boldsymbol{F}^p_m$. The possibility to split the deformation gradient in an elastic and plastic share gives rise to the introduction of a plastic intermediate configuration in the *co-/contravariant* domains for both metrics and stresses (cf. Figure 2).

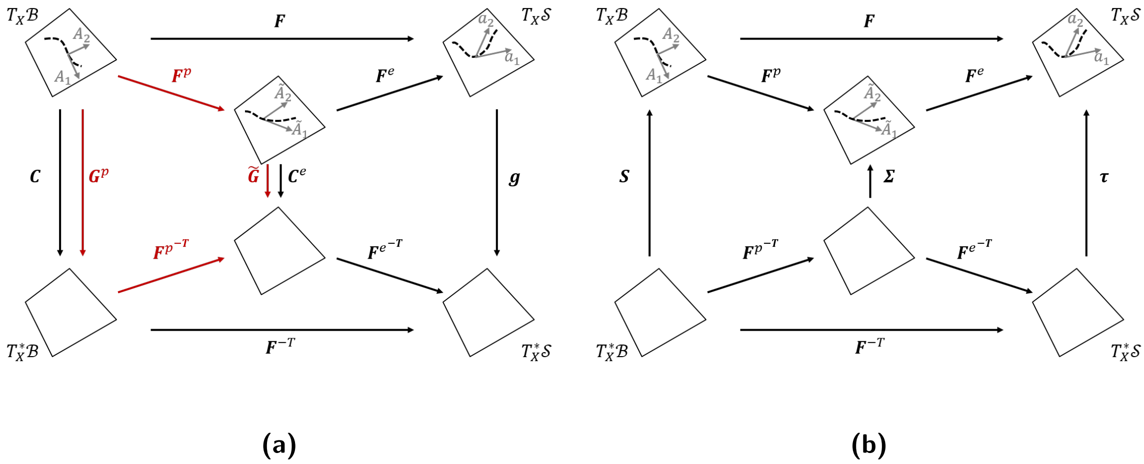


**Figure 2.** Schematic display of *co-/contravariant* domains for (a) metric and (b) stresses.

This allows in a way to seperate the plastic deformation, taking place in the matrix only, but at the same time is fully coupled onto the overall composite motion, and purely elastic behavior of all constituents. The normalized preferred directions $\tilde{\boldsymbol{A}}^n_i$ on the plastic intermediate configuration, deformed by purely plastic deformation, can be denoted as

$$\tilde{\boldsymbol{A}}^n_i = \frac{\boldsymbol{F}^p \boldsymbol{A}_i}{||\boldsymbol{F}^p \boldsymbol{A}_i||}. \tag{1}$$

### 1.2 Calculation of material stresses and moduli

The deformation of a continuous body is described by means of the right Cauchy-Green tensor, $\boldsymbol{C} = \boldsymbol{F}^T \boldsymbol{g} \boldsymbol{F}$, where $\boldsymbol{g}$ is the metric tensor in the Eulerian domain. The existence of a scalar potential

$$\Psi = \Psi^m \left(\boldsymbol{C}, \boldsymbol{G}^p, d_m\right) + \sum_{i=1}^{2} \Psi_i^r \left(\boldsymbol{C}, \boldsymbol{G}^p, \boldsymbol{A}_i, \boldsymbol{D}_i \left(\boldsymbol{C}\right)\right) \tag{2}$$

with $\Psi^m$ as the strain energy function of the matrix and $\Psi_i^r$ as the strain energy functions of the reinforcements, is assumed. Carefully note that, while $\Psi^m$ is independent of the preferred directions, $\Psi_i^r$ takes $\boldsymbol{A}_i$ as input, hence, introducing preferred directions, since the reinforcement structures are assumed to be initially transversely isotropic. Standard arguments yield the second Piola-Kirchhoff formulation for the tensions in both matrix and the reinforcements

$$\begin{aligned} \boldsymbol{S}^m &= 2\frac{\partial \Psi^m}{\partial \boldsymbol{C}} = \boldsymbol{S}^m \left(\boldsymbol{C}; \boldsymbol{G}^p, d_m\right), \\ \boldsymbol{S}_i^r &= 2\frac{\partial \Psi_i^r}{\partial \boldsymbol{C}} = \boldsymbol{S}_i^r \left(\boldsymbol{C}; \boldsymbol{G}^p, \boldsymbol{A}_i, \boldsymbol{D}_i\right). \end{aligned} \tag{3}$$

Analogously, the global tangent operator on the intermediate configuration can be expressed as

$$\begin{aligned} \mathbb{C}^m &= 4\frac{\partial^2 \Psi_i^r}{\partial \boldsymbol{C}^2} = \mathbb{C}^m \left(\boldsymbol{C}; \boldsymbol{G}^p, d_m\right), \\ \mathbb{C}_i^r &= 4\frac{\partial^2 \Psi_i^r}{\partial \boldsymbol{C}^2} = \mathbb{C}_i^r \left(\boldsymbol{C}; \boldsymbol{G}^p, \boldsymbol{A}_i, \boldsymbol{D}_i\right). \end{aligned} \tag{4}$$

Carefully note that stress and moduli concerning reinforcements are first calculated on the plastic intermediate configuration ($\boldsymbol{\Sigma}_i^r$, $\tilde{\mathfrak{c}}_i^r$), depending on $\tilde{\boldsymbol{A}}_i^n$, and then pulled back to the Lagrangeian setting. The damage tensor $\boldsymbol{D}_i$ in $\Psi_i^r$ takes into account a three-dimensional damage state depending on discrete failure mechanisms within the reinforcement structures, $i = 1, 2$, according to Naake et al. [5]. An elasto-viscoplastic matrix model incorporating an isotropic damage formulation for matrix damage ($d_m$) has been taken from Naake et al. [4]. With the assumption of no occuring plasticity in the matrix, *i.e.* $\boldsymbol{F}_m^p = \boldsymbol{1}$, reference configuration and plastic intermediate configuration will coincide. The overall second Piola-Kirchhoff stresses and according tangent moduli can finally be computed through the following expressions

$$\boldsymbol{S} = \boldsymbol{S}^m + \sum_{i=1}^{2} \boldsymbol{S}_i^r, \quad \text{and} \quad \mathbb{C} = \mathbb{C}^m + \sum_{i=1}^{2} \mathbb{C}_i^r. \tag{5}$$

## 2 Numerical example

The given fictitious example has the purpose to show the capability of the model to predict the change of orientation due to the application of an external load as well as residual deformation due to plasticity in the matrix. Damage effects have been neglected here for now. A one-layer-weave was subjected to an external shear load, as depicted in Figure 3. In Figure 3a the internal variables of initial material orientation is displayed in terms of $\boldsymbol{A}_i$. Due to a deformation $\bar{u}$ in $x_1$-direction the material deforms according to $\boldsymbol{F}$, thus yielding an orientation change, $\boldsymbol{a}_i$. Figure 4a shows the progression of the angle between the two preferred directions ($\gamma(t) = \sphericalangle(\boldsymbol{a}_1(t), \boldsymbol{a}_2(t))$) and Figure 4b the corresponding stress strain curve. The observable stiffening is caused by the progressive approach of the reinforcements rotating towards the loading direction, represented by decreasing values for $\gamma(t)$. Subsequent unloading reveals the residual change of the orientation due to plastic effects in the matrix.

## Conclusions and outlook

In the present work the principle of superposition of single constitutive models is used to model the overall behavior of a weave-reinforced thermoplastic composite. Instead of introducing direct

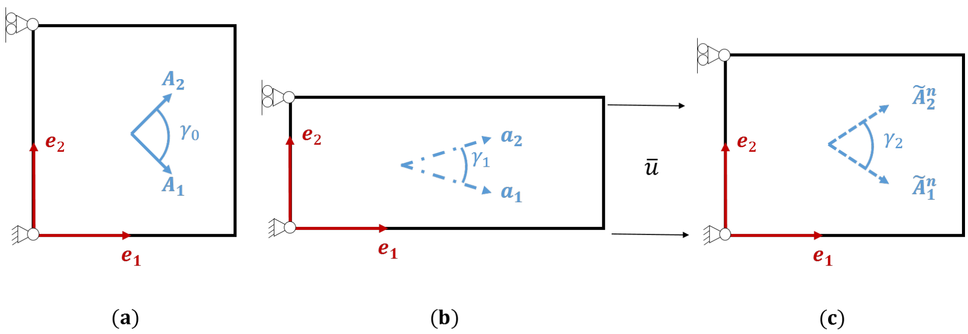


**Figure 3.** Schematic display of the loading path of a $\pm 45°$ off-axis woven composite. (a) Initial state with material orientation $\boldsymbol{A}_i$, (b) Fully loaded state with deformed preferred directions $\boldsymbol{a}_i$, (c) Unloaded state with resulting deformed orientation $\boldsymbol{a}_i^{res}$

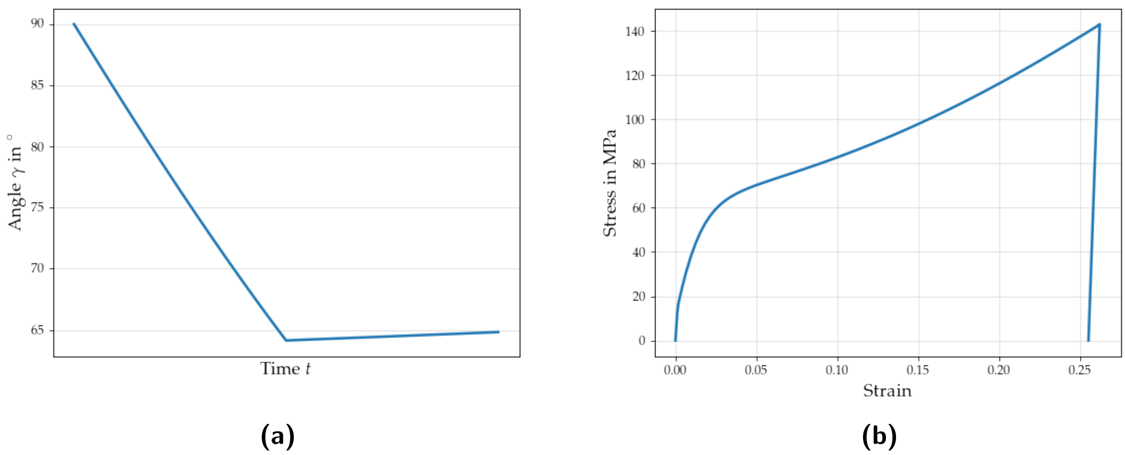


**Figure 4.** Display of numerical results of a $\pm 45°$ off-axis woven composite subjected to a shear load. (a) Progression of the inner angle of reinforcement orientation over time, $\gamma(t) = \sphericalangle(\boldsymbol{a}_1(t), \boldsymbol{a}_2(t))$. (b) Corresponding stress-strain curve.

formulations for plasticity and damage for the whole weave, this technique provides plasticity and a particular selection of damage criteria (for the matrix and single rovings) intrinsically. The model takes previously developed constitutive material models as an input and combines them by only using geometrical operations. However, any constitutive model for the constituents can be used. The presented numerical example shows that the geometrical effect of fiber rotation can be covered by the model.

A shortcoming of the presented model is that the actual weave type of the material and the corresponding interweaving is not respected in an entirely satisfactory way by only applying a simple superposition. This results in the necessity to introduce geometrically caused effects appearing in woven materials in a phenomenological manner (e.g. the locking angle, progressive stiffening of undulated rovings, their behavior under compression and delamination between rovings and matrix). These effects are the objective of future works and are neglected here.

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