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Cycle-by-cycle fatigue damage model for concrete

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Micro Abstract
Damage caused by stress concentrations in the complex mesoscopic geometry of concrete leads to continuous stress redistribution over the material's life time. The presented fatigue damage model captures this by resolving each load cycle in a cycle-by-cycle time integration. The model extends a static damage model to failure caused by the (time dependent) strain amplitudes and, thus, allows calibrating the majority of the material's parameters in static experiments.

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Introduction

Lifetime aspects including fatigue failure of concrete structures were traditionally only of minor importance due to the limited amplitude of the applied cyclic loads compared to the constant dead load. However, because of the growing interest in maxing out the capacities of concrete, its fatigue failure has become an important issue. Typical examples are offshore wind energy plants, which undergo extreme loading conditions of non-uniform amplitudes arising from wind and waves or fatigue loading of bridges with a steady increase of traffic load. However, a variety of interacting phenomena, such as the loss of prestress, the degradation due to chemical reactions or creep and shrinkage, influence the fatigue resistance. As a consequence, it is difficult to estimate the lifetime using only experimental techniques. Furthermore, failure due to cyclic loads is generally not instantaneous, but characterized by a steady damage accumulation. Therefore, a reliable numerical model to predict the performance of concrete over its lifetime is required.

1 Fatigue damage model

Fatigue behavior is usually divided into two categories, high-cycle fatigue and low-cycle fatigue. High-cycle fatigue involves the application of a large number of cycles ($\approx 10^6$) at low load levels. Many numerical models employ Paris law [7]. A damage equation is formulated including the number of cycles N as a model parameter [8, 9]. This approach cannot capture stress redistributions during the loading history.

Low-cycle fatigue aims at capturing few loading cycles at high amplitudes. A stress-strain relationship is integrated in a cycle-by-cycle integration, requiring $O\,(10)$ time steps by cycle. One approach, followed in this paper, is Marigo's formulation [4]. A static damage model - that only accumulates damage if its driving variable reaches a historic maximum - is extended to allow damage growth well below this value. This approach is little invasive and allows straightforward coupling in a multiphysics context, e.g. including temperature strains, creep phenomena or plasticity.

1.1 Definition of a static local damage model

The local damage model is a combination of a linear elastic model and the damage variable ω, ranging from $\omega = 0$ (virgin material) to $\omega = 1$ (complete loss of stiffness)

$$\nabla \cdot \boldsymbol{\sigma} = \nabla \cdot ((1 - \omega(\kappa))\, \mathbb{C} : \boldsymbol{\varepsilon}) = \mathbf{0} \tag{1}$$

with the isotropic stiffness tensor $\mathbb{C}$, the Cauchy stress $\boldsymbol{\sigma}$ and the infinitesimal strain $\boldsymbol{\varepsilon}$. The damage is driven by the strain-like history variable κ through the damage law [5, 6]

$$\omega(\kappa) = \begin{cases} 0 & \kappa < \kappa_0 \\ 1 - \frac{\kappa_0}{\kappa} \exp\left(-\frac{f_{ct}}{g_f}(\kappa - \kappa_0)\right) & \text{otherwise} \end{cases} \tag{2}$$

with the material parameters $k_0 = f_t/E$, Young's modulus E, tensile strength f_t and the fracture energy parameter g_f. The Karush-Kuhn-Tucker conditions

$$\dot{\kappa} \geq 0, \quad \varepsilon_{\mathrm{eq}} - \kappa \leq 0, \quad \dot{\kappa}\,(\varepsilon_{\mathrm{eq}} - \kappa) = 0 \tag{3}$$

determine κ by a scalar equivalent strain measure $\varepsilon_{\mathrm{eq}}$. The strain-based modified von Mises definition [1] is used here. It is capable of capturing the difference of tensile strength to compressive strength of concrete.

Equation (3) is transformed equivalently into the evolution equation

$$\dot{\kappa} = \begin{cases} \dot{\varepsilon}_{\mathrm{eq}} & \text{if } \varepsilon_{\mathrm{eq}} = \kappa \\ 0 & \text{otherwise,} \end{cases} \tag{4}$$

meaning that κ only grows (and damage ω increases) if the equivalent strains exceed their historic maximum. This relates to the *static loading* case in fig. 1.

1.2 Extension to cyclic loading

Equation (4) is extended to allow damage growth below the historic maximum $\kappa = \varepsilon_{\mathrm{eq}}$.

$$\dot{\kappa} = \begin{cases} \dot{\varepsilon}_{\mathrm{eq}} & \text{if } \varepsilon_{\mathrm{eq}} = \kappa \\ |\dot{\varepsilon}_{\mathrm{eq}}| f & \text{otherwise} \end{cases} \tag{5}$$

The scaling function f has to be chosen empirically. Here it is defined as

$$f(\boldsymbol{\sigma}) = A\, \frac{\left\langle \sigma_{\mathrm{eq}}(\boldsymbol{\sigma}) - \sigma_\infty \right\rangle_+}{f_t}, \tag{6}$$

where σ_{eq} is the Rankine stress norm. The combination of the fatigue limit σ_∞ and the Macaulay brackets $\langle\rangle_+$ prevents damage growth below σ_∞. The scaling parameter A allows further calibration of the model. Equation (5) is discretized in each global time step n. An implicit Euler backward requires a solve for κ for each integration point. This is avoided by using the explicit Euler scheme.

2 Integration point results

Uniaxial stress/strain curves are plotted for various loading cases in fig. 1. A monotonic increase of the strain results in the *static loading*. Stress increases with the elastic stiffness until the tensile strength f_t is reached. The post-peak behavior is controlled by the damage law (eq. (2)). The *strain controlled* curve is created with two cyclic loading phases. The very first cycle follows the static curve. Each subsequent unloading/loading cycle increases the damage and reduces the materials stiffness. During the transition to the second loading phase (cyclic loading at a higher

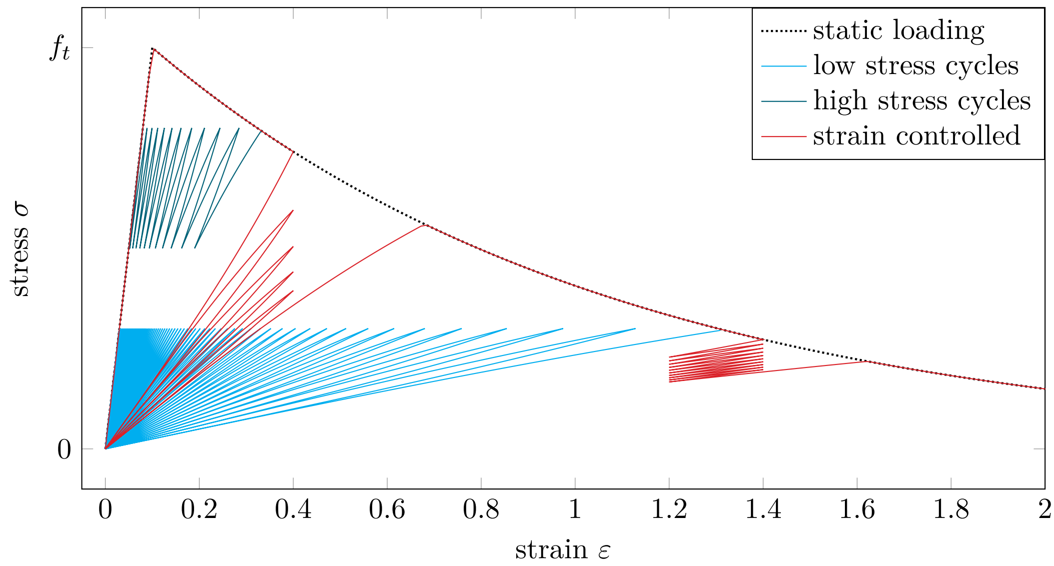

Figure 1. Stress and strain controlled low-cycle fatigue experiments.

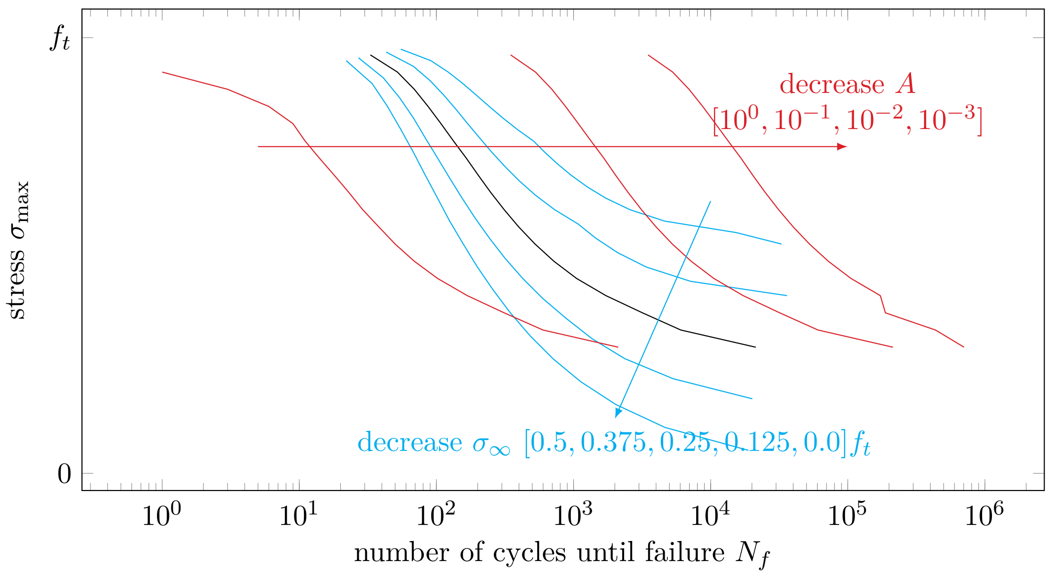

Figure 2. Parameter study based on Wöhler lines with cyclic loading between 0 and $\sigma_{\max}$

mean strain), the static limit is reached again. Further loading, again, reaches the static envelop curve and follows it.

The two stress controlled experiments with the same stress amplitude at different mean stresses are also shown. Once the static curve is reached, the desired stress can no longer be reached, which corresponds to material failure. The definition of the scaling function f (eq. (6)) includes the current stress $\boldsymbol{\sigma}$. Thus, the material endures less loading cycles at high stresses compared to loading at a lower mean stress.

The fatigue behavior can be characterized by $\sigma - N$ curves, also known as Wöhler curves that are shown in fig. 2. The endurable number of stress cycles N_f for a cyclic sinusoidal loading between 0 and $\sigma_{\max}$ is shown. Cycles below the endurance stress σ_∞ do not lead to material failure. Additionally, this parameter influences the slope of Wöhler curves. The scaling parameter A shifts the curves.

3 Computational cost

The application of the cycle-by-cycle integration to high-cycle fatigue requires sophisticated time integration schemes to handle the computational cost. Temporal multi-scale schemes [2, 3] or *jump-in-cycle* techniques [8] assume an almost linear evolution of the damage variable over a wide range of cycles. The damage increment of a few explicitly integrated cycles is extrapolated to a large number of cycles. The computational cost can be reduced to a fractional amount of the full cycle-by-cycle analysis and the approach becomes feasible.

Conclusions

The presented fatigue model is a little invasive extension of an isotropic static damage model. The evolution equation of the damage driving variable is enhanced to allow damage growth below the static limit. Since the damage law remains untouched, most of the models parameters can be calibrated in static experiments.

Wöhler curves on an integration point level are obtained from a cycle-by-cycle time integration of the model. As shown in the parameter study, the two fatigue parameters allow calibration to experimental data.

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