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Gramian-Based Actuator Placement for Static Load Compensation in Adaptive Structures

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Micro Abstract
The compensation of static loads aims at reducing stresses or displacements by applying energy to an adaptive structure. The performance in static adaption significantly depends on the location of the actuators. We present a method for optimal actuator placement regarding static adaption using a Gramian-based cost function. The method is demonstrated by means of a numerical model of an adaptive truss structure. Results indicate the method's effectiveness to promote actuator placement.

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Introduction

The concept of adaptivity enables the design of lightweight structures beyond the current state-of-the-art through continuous adaptation of the load-bearing behavior to variable load conditions, thereby offering a great potential for material savings in the construction sector [6,8]. For this purpose, adaptive structures are equipped with actuators generating the required control forces [2]. The adaptive capability crucially depends on the number and arrangement of the actuators. While a greater number generally increases the adaptive capability, the cost and complexity of the system are increased at the same time. Actuator placement in structural design aims at solving this trade-off by a careful selection of a limited number of well-placed actuators.

There are a number of methods for actuator placement proposed in literature. In [1], an optimization-based placement method concerning active damping of linear structural systems under dynamic load conditions is introduced. A method for actuator placement in the context of static load compensation in truss structures is proposed in [5,7]. It determines an efficiency value for each actuator location under the assumption of quasi-static loads. The sorted efficiency gives the order of the actuators' importance.

In this paper, a new method for actuator placement regarding static load compensation in linear structural systems under quasi-static loads is introduced. It is based on a scalar performance metric describing the achievable load compensation for a certain actuator configuration. The optimal actuator configuration is then found by solving a combinatorial optimization problem. The remainder of this paper is organized as follows: The next section describes the modeling of a mechanical structure in the stationary case. In Section 2, the proposed method is introduced in detail. In Section 3, the method is demonstrated by means of a scale model of an adaptive high-rise truss structure.

1 System Modeling

In the stationary case, the deformation of a linear mechanical structure is governed by

$$\boldsymbol{K}\boldsymbol{q} = \boldsymbol{B}\boldsymbol{u} + \boldsymbol{E}\boldsymbol{z}, \tag{1}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ are the degrees of freedom. $\boldsymbol{K} \in \mathbb{R}^{n \times n}$ is the stiffness matrix, which can be obtained e.g. by finite element analysis. The matrices $\boldsymbol{B} \in \mathbb{R}^m$ and $\boldsymbol{E} \in \mathbb{R}^{n \times l}$ are the input matrices of the control inputs $\boldsymbol{u} \in \mathbb{R}^m$ and disturbances $\boldsymbol{z} \in \mathbb{R}^l$, respectively. The system's output $\boldsymbol{y} \in \mathbb{R}^p$ is given by

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{q}, \tag{2}$$

where the output matrix $\boldsymbol{C} \in \mathbb{R}^{p \times n}$ is governed by the type of output. Depending on the application, it can reflect e.g. displacements or stresses in the structure.

2 Actuator Placement

The actuator placement problem can be stated as follows: Given a limited number of actuators and a fixed set of actuator candidates, select that combination of actuators achieving the best load compensation. In order to do so, a metric assessing the quality of a certain actuator configuration is introduced in the next section. Subsequently, the actuator placement is formulated as a combinatorial optimization problem and solved using an heuristic approach in Section 2.2.

2.1 Performance Metric

In general, static load compensation aims at reducing the difference between a desired output $\boldsymbol{y}_\mathrm{d}$ and the current output $\boldsymbol{y}$ under arbitrary disturbances $\boldsymbol{z}$ by suitable choice of a control input $\boldsymbol{u}$. Therefore, the minimization of the Euclidean norm of the output error $\boldsymbol{e} = \boldsymbol{y}_\mathrm{d} - \boldsymbol{y}$ is pursued. Without loss of generality, $\boldsymbol{y}_\mathrm{d} = 0$ is assumed in the following. Solving (1) for $\boldsymbol{q}$ and plugging the result into (2) yields

$$\boldsymbol{e} = \boldsymbol{C}\boldsymbol{K}^{-1}(\boldsymbol{B}\boldsymbol{u} + \boldsymbol{E}\boldsymbol{z}). \tag{3}$$

For a given disturbance $\boldsymbol{z}$, the error norm is minimized by the least-square solution for the optimal input

$$\boldsymbol{u}^* = \arg\min_{\boldsymbol{u}} \|\boldsymbol{e}\|_2^2 = -(\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{B})^+\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{E}\boldsymbol{z}, \tag{4}$$

where $(\cdot)^+$ denotes the pseudo-inverse. Substituting $\boldsymbol{u}^*$ for $\boldsymbol{u}$ in (3), the minimal error is

$$\boldsymbol{e}^* = \boldsymbol{H}\boldsymbol{z}, \quad \text{with} \quad \boldsymbol{H} = \Big((\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{B})(\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{B})^+ - \boldsymbol{I}\Big)\boldsymbol{C}\boldsymbol{K}^{-1}\boldsymbol{E} \tag{5}$$

yielding the minimal squared Euclidean error norm

$$\|\boldsymbol{e}^*\|_2^2 = \boldsymbol{z}^\intercal \boldsymbol{W}\boldsymbol{z}, \quad \text{with} \quad \boldsymbol{W} = \boldsymbol{H}^\intercal\boldsymbol{H}. \tag{6}$$

As can be observed from (6), the numerical value of the minimal error norm depends on the specific disturbance $\boldsymbol{z}$, which is generally unknown. However, the Gramian matrix $\boldsymbol{W}$ allows to quantify the achievable minimal error norm independent of a specific disturbance $\boldsymbol{z}$ by regarding arbitrary but normalized disturbances on the unit hyper-sphere $\{\boldsymbol{z} : \|\boldsymbol{z}\|_2 = 1\}$. For example, in this case the maximum eigenvalue $\lambda_{\max}(\boldsymbol{W})$ is proportional to the maximum squared error norm. Analogously, the trace $\mathrm{tr}(\boldsymbol{W})$ is proportional to the mean squared error norm. Note that a similar argument is used in literature for the derivation of Gramian controllability and observability metrics [4].

2.2 Optimization

Based on the performance metric derived in the previous section, the actuator problem can now be formulated as an optimization problem. Given is a set of actuator candidates $C = \{\boldsymbol{b}_1, \boldsymbol{b}_2, \ldots, \boldsymbol{b}_m\}$ with $|C| = m$, where $\boldsymbol{b}_i$ is the input matrix of the i^{th} candidate. The aim of the actuator placement is to select the subset $S \subseteq C$ with cardinality $|S| = k \leq m$, minimizing the cost function $J(S)$, formally

$$(S^*, k^*) = \arg \min_{S \subseteq C, 0 < k \leq m} J(S). \tag{7}$$

S^* denotes the optimal set and $|S^*| = k^*$ the optimal amount of actuators. $J(S)$ can be computed for a given set S by using the performance metric derived in Section 2.1. For the computation of the Gramian matrix $\boldsymbol{W}(S)$, the respective input matrix $\boldsymbol{B}(S)$ is assembled from the input matrix columns specified by S.

The problem (7) belongs to combinatorial optimization. The calculation of the optimal solution by enumeration is infeasible even for small problems as the number of all possible combinations is $\sum_{k=1}^{m} \binom{m}{k}$. Under certain conditions, the optimal solution can be stated directly. Otherwise, several heuristics are given in literature to solve such problems. Of these, greedy algorithms present a remarkably simple and effective class. The reverse-delete algorithm was firstly proposed in [3]. This iterative algorithm starts with the full set of actuators $S_0 \leftarrow C$. In every iteration step, the algorithm computes the increase in the cost function $\Delta(S_i, s) = J(S_i \backslash \{s\}) - J(S_i)$ when taking away one actuator s at a time for every remaining actuator $s \in S_i$. Then, the actuator with the minimal increase of the cost function is removed from the solution set, i.e. $S_{i+1} \leftarrow S_i \backslash \{\arg\min \Delta(S_i, s) | s \in S_i\}$. This procedure is repeated until the desired number of actuators is obtained or there are no more actuators left in the solution set.

3 Numerical Example

The proposed method for the use of actuator placement is demonstrated in a numerical example of a 1:18 scale model of a high rise truss structure, which is separated in $N = 5$ partitions with mass $m = 5\,\mathrm{kg}$ each. Such a partition consists of four vertical beams, one in each corner, and a diagonal bracing on each side with two further beams. The truss structure is modeled with the finite element method, where the Euler-Bernoulli theory is used to compute the element stiffness matrices. Further model parameters are the truss width $w = 0.26\,\mathrm{m}$ and height $h = 0.4\,\mathrm{m}$ of one partition, and the stiffness of the vertical ($k_\mathrm{v} = 0.02\,\mathrm{N/m}$) and the diagonal ($k_\mathrm{d} = 0.01\,\mathrm{N/m}$) elements. Locations for actuators are all vertical and diagonal elements, resulting in $m = 60$ actuator candidates. The output matrix $\boldsymbol{C}$ is defined so that $\boldsymbol{y}$ contains the nodal displacements in the translational directions. Disturbances are assumed to be arbitrary, subsequently $\boldsymbol{E} = \boldsymbol{I}$ with $\boldsymbol{I}$ being the identity matrix. The cost function is the norm of the average nodal displacement error, i.e. the trace of $\boldsymbol{W}$:

$$J(S) = \mathrm{tr}(\boldsymbol{W}(S)). \tag{8}$$

$J(S) = 0$ means that the norm of the output error is completely compensated and adding further actuators would not achieve any improvement. Usually, a high number of actuators is necessary to achieve this.

The normalized cost function $J_\mathrm{norm}(S) = (J(C) - J(\emptyset))^{-1}(J(C) - J(S))$ is introduced to simplify the interpretation of the results, where $J(C)$ is the cost function value for the complete set, i.e. using all possible actuators, and $J(\emptyset)$ is the cost function value, not using any actuation at all. In Figure 1, the result of the actuator placement is depicted. In (a) the normalized cost function $J_\mathrm{norm}(S)$ is plotted over the number of actuators. A relatively low number of actuators already achieves a good result. In Figure 1 (b) the actuators are colored according to the order of selection. A light color indicates an early selection. Twelve actuators $k^* = 12$ are sufficient to reach $> 99\,\%$ of the maximum value $J(C)$ as the gray line indicates in Figure 1 (a). The actuator distribution is shown in Figure 1 (c). The actuators are allocated over the full length of the structure with an aggregation in the second partition. This amount of actuators can be accepted with regarding system complexity and cost.

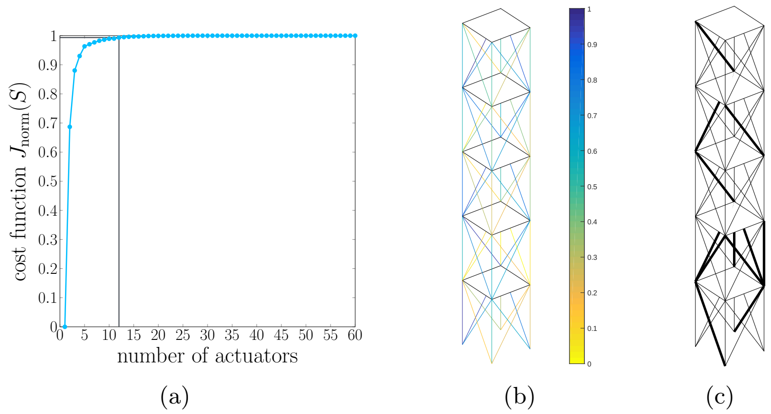

Figure 1. Optimization result with $\mathrm{tr}(\boldsymbol{W}(S))$. (a) Normalized cost function $J_{\text{norm}}(S)$ over the number of actuators. Grey line: Solution, which reaches $> 99\,\%$ of $J_{\text{norm}}(C)$ the first time and $k^* = 12$ actuators are needed. (b) Visualization of the actuator selection, high value/yellow indicates an early selection. (c) Binary decision of selected actuator set S^*.

Conclusions

This paper introduces a method for actuator placement in the context of static load compensation. A Gramian-based performance metric is derived, describing the output error norm independent of specific disturbances. It serves as a basis for an optimization heuristic, specifically the reverse delete greedy-algorithm to determine optimal actuator positions. The numerical results indicate that the solution is very close to the optimum while maintaining a reasonable number of actuators. Further research can be dedicated to the examination of other performance metrics and alternative optimization procedures. Furthermore, the combination of actuator placement methods for static and dynamic load compensation has to be considered.

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