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A phase field model for materials with anisotropic fracture resistance

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Micro Abstract
Directional dependency of the fracture resistance, which is observed for a wide range of materials, requires the integration of anisotropic behavior in approaches for crack growth simulations. The gradient term in the energy functional of a phase field model for isotropic brittle fracture is enhanced accounting for an anisotropic material resistance. Results of crack path simulations for different samples are presented to show the accuracy of the proposed model.

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Introduction

Anisotropy of materials is an important property, which has to be considered within the development and realization of engineering structures. The directional dependency of mechanical properties can either be desired for an optimization of a specific load path, which is achieved e.g. by changing the fibre orientation of a composite, or occur as a consequence of several manufacturing processes like rolling or extruding. A further differentiation of the anisotropic behaviour of a material has to be made between the elastic anisotropy and the anisotropy of the fracture resistance. Whereas the elastic anisotropy effects the reaction of the material to a certain load and is responsible – with view on continuum mechanics – for additional elastic constants of the elasticity tensor, the anisotropy of the fracture resistance influences the path of a growing crack caused by external loads.
Phase field fracture models provide a convenient framework for the prediction of crack paths. Within these models, an additional field variable is introduced to represent the crack field. This field is an approximation of the sharp crack with a continuous transition zone between broken and unbroken material. The evolution equation for the crack field is based on Bourdin's [3] regularized formulation of the variational model for brittle fracture proposed by Francfort and Marigo [5]. This model addresses Griffith's theory, in which a crack propagates once the energy release per unit crack surface area is balanced by the increase of surface energy caused by infinitesimal crack growth. As an extension, it is postulated that a global minimum of the total energy, which is the sum of bulk and surface energies, always appears within a loaded sample.A comparison of several phase field fracture models is provided in [1].
Since the surface part of the total energy depends on the critical energy release rate $\mathcal{G}_c$ and hence on a material constant, anisotropic crack growth can be included by assuming $\mathcal{G}_c$ to be a function of the crack propagation direction. This approach for anisotropic material behaviour was introduced by Hakim and Karma [7] and by Li et al. [11] in the context of phase field models. Another way is to leave $\mathcal{G}_c$ constant and rather modify the gradient term of the surface part in the energy functional. This second approach is studied within this work.

1 Directional fracture resistance

The directional dependency of the resistance against brittle crack extension in metallic materials is effected by the forming of textures during manufacturing. An elongation of the grains under a certain direction arises as a consequence of rolling or extrusion processes (see e.g. [9]). Two different kinds of crack propagation are characterized in the context of brittle fracture. Dependent on the relative strength of the grain boundaries, a crack grows either along these boundaries (intercrystalline) or through the grains (transcrystalline). However, there is a preferred direction of the crack path for both types since the energy needed for crack extension is minimized if less grain boundaries have to be passed in the case of transcrystalline crack growth, or by choosing the less tortuous path in case of intercrystalline growth (see e.g. [8]).

2 Phase field model for anisotropic crack growth

2.1 Isotropic phase field model

The basic model used for the performed simulations was proposed by Kuhn and Müller in [10]. This phase field fracture model contains a regularized expression for the total energy introduced by Bourdin [3]. Beside the bulk part, this total energy consists of a surface term accounting for the energy required for the irreversible processes associated with the formation of a crack.

$$E(\boldsymbol{\varepsilon}, s) = \int_\Omega \underbrace{(g(s)+\eta)\frac{1}{2}\boldsymbol{\varepsilon} : [\mathbb{C}\,\boldsymbol{\varepsilon}]}_{\psi_e} + \underbrace{\mathcal{G}_c\left(\frac{(1-s)^2}{4\epsilon} + \epsilon|\nabla s|^2\right)}_{\psi_c} dV. \tag{1}$$

The field variable representing a crack in this model is s, which takes the value 1 for intact material and 0 in case of a crack. The first term in the integral of (1) is the strain energy density ψ_e, in which $\boldsymbol{\varepsilon}$ is the linearized strain tensor and $\mathbb{C}$ is the possibly anisotropic stiffness tensor. The $:$ operator denotes the double dot product of two second order tensors. The decrease of strain energy coming along with the loss of stiffness in non intact material is modelled by the degradation function $g(s)$. In the original formulation by Bourdin a quadratic function $g(s) = s^2$ is used. As an alternative a cubic degradation function was introduced by Borden et al. in [2]. The parameter η ensures a residual stiffness in case of $s = 0$ to avoid numerical difficulties. The crack energy density ψ_c is the product of the critical energy release rate $\mathcal{G}_c$ and a crack density functional, where the parameter ϵ controls the width of the smooth transition zone between a crack and the undamaged material. To obtain the time evolution of the crack field s, the generalized Ginzburg-Landau equation (see e.g. [6]) with (1) as energy functional is applied. With the functional derivative $\delta\psi/\delta s$ the evolution equation for s is

$$\dot{s} = -M\frac{\delta\psi}{\delta s} = -\frac{M}{2}\left[g'(s)\boldsymbol{\varepsilon} : [\mathbb{C}\,\boldsymbol{\varepsilon}] - \mathcal{G}_c\left(4\epsilon\Delta s + \frac{1-s}{\epsilon}\right)\right]. \tag{2}$$

In this equation, the scalar mobility M acts as a control parameter to approach the state of equilibrium, specified by $\delta\psi/\delta s = 0$. Accordingly, quasi static conditions are approximated by M approaching infinity.

2.2 Extension to model anisotropy of the fracture resistance

The energy density needed within the process zone for the formation of a crack is a material dependent property and is quantified by the critical energy release rate $\mathcal{G}_c$. A common way to introduce anisotropic behaviour related to a directional dependency of the fracture resistance is to assume $\mathcal{G}_c$ as function of the potential crack growth direction, i.e. $\mathcal{G}_c = \mathcal{G}_c(\varphi)$, where φ indicates the direction with respect to a specified axis. This approach has been studied within [7] and [11] in the context of phase field models for fracture. However, a simple alternative is to leave $\mathcal{G}_c$ constant and introduce additional material parameters accounting for the anisotropy.

The crack density functional of (1) consists of a local part and a non-local term proportional to the squared norm of the spatial gradient of the crack field variable s. This squared norm can be rewritten as

$$|\nabla s|^2 = \nabla s \cdot (\mathbf{1} \nabla s), \tag{3}$$

where $\mathbf{1}$ is the second order identity tensor and $\cdot$ denotes the dot product of two vectors. The key of the modification implemented within this work is the substitution of the identity in (3) by a diagonal tensor Φ of the form:

$$\Phi = \begin{pmatrix} 1+\alpha & 0 \\ 0 & 1-\alpha \end{pmatrix} \tag{4}$$

for the two-dimensional case. Note that by the definition above $\mathrm{tr}(\Phi) = \mathrm{tr}(\mathbf{1}) = 2$. With this minor modification, the energy added by crack extension is now weighted for each spatial direction. To satisfy material properties as outlined in Section 1, a direction prone to crack extension can now be specified by means of the parameter α. With the modified gradient term the evolution equation (2) for the crack field s becomes

$$\dot{s} = -\frac{M}{2}\left[g'(s)\boldsymbol{\varepsilon} : [\mathbb{C}\,\boldsymbol{\varepsilon}] - \mathcal{G}_c\left(4\epsilon\nabla\cdot(\Phi\nabla s) + \frac{1-s}{\epsilon}\right)\right]. \tag{5}$$

3 Numerical examples

3.1 Inclined crack under uniaxial tensional load

As a first numerical example, the extension of an inclined crack under uniaxial tension is simulated (all simulations performed with dimensionless quantities). The load case, which is schematically shown in Fig. 1(a) causes a mixed mode loading at the initial crack tip. Accordingly, the crack deflection angle φ is a function of the initial crack angel γ.

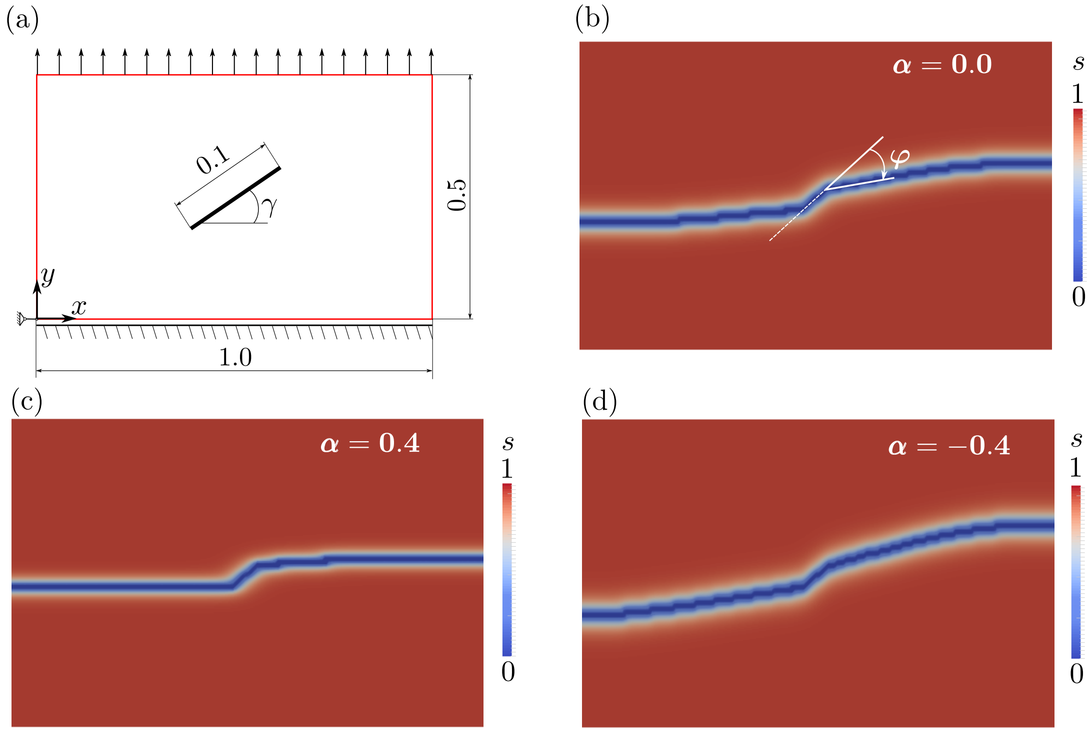

Figure 1. Load case (a) and contour plots of the crack field obtained by simulations ($\nu = 0.25$, $\epsilon = 0.005$, $M = 10$) for different values of the parameter α: (b) $\alpha = 0$, (c) $\alpha = 0.4$, (d) $\alpha = -0.4$

For this simulation γ was set to 45°. The crack field obtained without weighting of the gradient components (Fig. 1(b)) reveals a deflection angle of about 35°. Figures 1(c) and (d) show the crack field obtained by simulations, where the parameter α was set to 0.4 and -0.4, respectively. The contour plot of Fig. 1(c) shows a nearly horizontal crack extension ($\varphi \approx 42°$), which is caused by higher costs of gradients in x-direction. On the contrary, Fig. 1(d) reveals a decrease of the crack deflection angle ($\varphi \approx 30°$) due to the higher weighting for the gradient in y-direction.

3.2 Plates with hole

For the second example, the proposed anisotropic phase field model is applied to experiments performed within a work of Judt and Ricoeur [9]. In this work rolled aluminium sheets which reveal a directional dependency of the fracture resistance are investigated. Therefore, tensile tests of plate specimens with an initial crack were performed. Results of the crack growth experiments are compared with simulations. A main focus is on how the level of the anisotropy effects the crack path. The anisotropy is thereby quantified by the factor $\chi = K_{Ic}^{TD}/K_{Ic}^{RD}$ containing the fracture toughness in transverse$^{(TD)}$ and rolling$^{(RD)}$ direction, respectively. Transient numerical simulations including crack tip shifting and automatic remeshing were performed to obtain the crack path in [9]. Within these simulations the criterion of maximum energy release rate [4] was applied to estimate the direction of the crack extension. According to this approach, the crack grows in the direction providing the maximum energy release rate $\mathcal{G}(\varphi)$. To include anisotropy of the fracture resistance the critical energy release rate $\mathcal{G}_c(\varphi)$ was interpolated between $\mathcal{G}_c^{TD}$ and $\mathcal{G}_c^{RD}$. The crack was incrementally enlarged in the direction, which maximizes the quotient $\mathcal{G}(\varphi)/\mathcal{G}_c(\varphi)$. Figure 2(a) shows a detail of an investigated specimen with calculated crack paths indicated for three different factors χ. This specific example is used to verify the proposed phase field model. Figures 2(b-d) show the crack field obtained by phase field simulations for three different values of the parameter α.

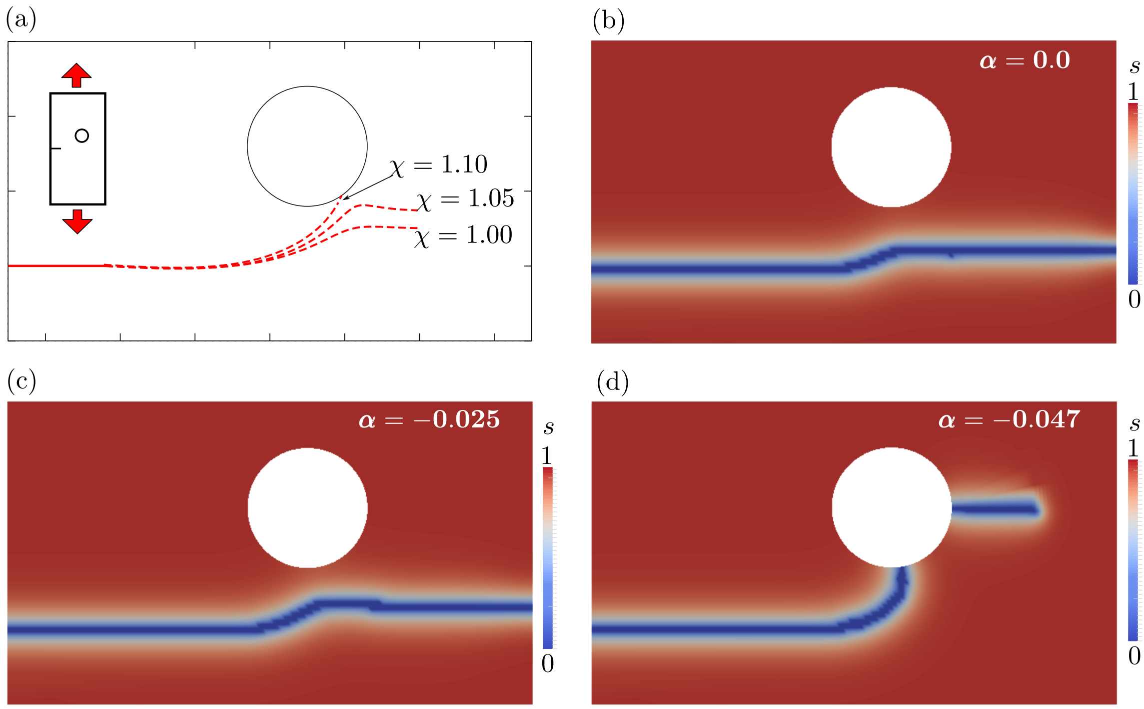

Figure 2. Comparison of crack paths obained by the phase field model ($\nu = 0.33$, $\epsilon = 0.006$, $M = 10$) with results of [9]: (a) results of [9], (b) $\alpha = 0.0$, (c) $\alpha = -0.025$, (d) $\alpha = -0.047$.

Within the modified phase field model an anisotropy regarding the fracture resistance is solely quantified by the ratio of the coefficients of Φ. Accordingly, this ratio can be connected to the ratio of the fracture resistance in different spatial directions χ by $\alpha = \frac{1-\chi}{1+\chi}$. The α-value for the three samples of Fig. 2(b-d) were calculated this way using the particular χ-values leading to the different crack courses illustrated in Fig. 2(a). If no anisotropy is considered ($\alpha = 0$), the crack deflects from the horizontal axis and extends upwards as it passes the holes left contour edge. Beyond the center of the hole the crack grows in horizontal direction again. The resulting vertical offset of the crack path is larger if α is set to -0.025. For $\alpha = -0.047$ the crack does not pass the hole, but rather grows directly in its direction.

Conclusions

The effect of an anisotropic fracture resistance on the crack path is realized by a modification of the gradient term of an existing phase field model for fracture. The behaviour of the enhanced model is illustrated by two numerical examples. The weighting in the gradient term of the crack energy functional leads to a different crack path as compared to the isotropic case. The simulations show that a crack extension in the direction of a lower fracture resistance is preferred within the crack field evolution. Further proof of the model is made by the comparison to an alternative crack path prediction method. Even if the three simulated crack fields of the second example do not perfectly coincide with the results of [9], it can however be confirmed that the trend of the crack path obtained by the phase field simulation is consistent with regard to the effect of an anisotropic fracture resistance.

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