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Simulation and optimal control of dielectric elastomer actuated systems

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Micro Abstract
This contribution introduces different modelling techniques and optimal control approaches for humanoid structures that are actuated by dielectric elastomers. Dielectric elastomers, also known as artificial muscles, belong to the group of smart materials. When excited with an electrical voltage, the elastic material contracts noiselessly, allowing for smooth motion of the actuated system.

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Introduction

Dielectric elastomer actuators (DEAs) are composed of an elastic dielectric material that is sandwiched between two compliant electrodes, as illustrated in Figure 1. When the electrodes are charged by applying an electric potential to them, charges with opposite signs attract each other, leading to a contractive force also known as electrostatic pressure [4]. When several DEA cells are stacked on top of each other, resulting in a pile-up configuration, the electrostatic pressure provides macroscopically useful displacements [1]. Stacked DEAs are also referred to as artificial muscles, because they bear analogy to the behaviour of human muscles in terms of contracting in length direction when stimulated. The idea of using artificial muscles as sophisticated actuators offers a broad variety of potential applications. The elastic structure acts as an energy storage and allows for dynamic motion of robots and safe human interaction. However, the use of elastic actuators is also accompanied by new control challenges. Advanced control strategies need to avoid undesirable oscillations, bring the system as quickly as possible into its steady state and follow prescribed trajectories as close as possible.

In this work, two different DEA models are presented. The first model is based on a general three-dimensional field theory of electromagnetic forces in deformable continua with arbitrary geometry. This model is very powerful but also computationally very costly and hence suitable only for forward dynamic simulations. The second approach exploits symmetries, regularities and predicted behaviour of the muscles, leading to a so called lumped parameter model, where spatially discrete configuration variables condense the complex physical relationships. As this model reduces the computational cost drastically, it can be utilised for optimisation tasks which is illustrated in a numerical example.

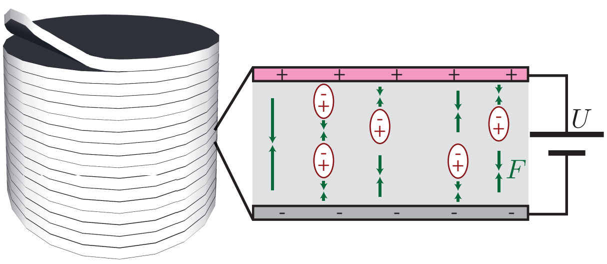

Figure 1. Functional principle of a dielectric elastomer actuator.

1 Muscle Model

The electromechanically coupled dynamical system within the actuator $\mathcal{B}_0$ and on the actuator surface $\partial\mathcal{B}_0$ with surface normal $\boldsymbol{N}$ can be written as

$$\nabla_{\boldsymbol{X}} \cdot \boldsymbol{P}^{\text{tot}} + \boldsymbol{b}_0^{\text{mech}} = \rho_0 \ddot{\boldsymbol{x}} \qquad \nabla_{\boldsymbol{X}} \cdot \boldsymbol{D} = 0 \qquad \text{in} \quad \mathcal{B}_0 \tag{1a}$$

$$\boldsymbol{P}^{\text{tot}} \cdot \boldsymbol{N} = \overline{\boldsymbol{T}} \qquad \boldsymbol{D} \cdot \boldsymbol{N} = -\overline{Q} \qquad \text{in} \quad \partial\mathcal{B}_0 \tag{1b}$$

where $\boldsymbol{b}_0^{\text{mech}}$ is a mechanical volume force, ρ_0 is the material density, $\ddot{\boldsymbol{x}}$ the acceleration of a material point, $\overline{\boldsymbol{T}}$ a mechanical surface traction, $\boldsymbol{D}$ the electric displacement and $\overline{Q}$ an electric charge density. The total first Piola-Kirchhoff stress $\boldsymbol{P}^{\text{tot}}$ is composed such that

$$\boldsymbol{P}^{\text{tot}} = \boldsymbol{P}^{\text{ela}} + \boldsymbol{P}^{\text{vis}} + \boldsymbol{P}^{\text{ele}}, \tag{2}$$

where $\boldsymbol{P}^{\text{ela}}$ is the pure mechanical stress, $\boldsymbol{P}^{\text{vis}}$ covers viscous contributions and $\boldsymbol{P}^{\text{ele}}$ is the Maxwell stress tensor that originates from the applied electric field and associated polarisation effects within the material [6].

Constitutive relationships are specified via an electromechanically coupled hyperelastic material approach. Assuming the deformation gradient $\boldsymbol{F}$ and the electric field $\boldsymbol{E}$ are independent variables, the potential energy density Ω might take the form

$$\Omega(\boldsymbol{F},\boldsymbol{E}) = \underbrace{\frac{\mu}{2}[\boldsymbol{C}:\boldsymbol{1}-3] - \mu\ln(J) + \frac{\lambda}{2}[\ln(J)]^2}_{\text{Neo-Hooke}} + \underbrace{c_1\boldsymbol{E}\cdot\boldsymbol{E}}_{\text{electric}} + \underbrace{c_2\boldsymbol{C}:[\boldsymbol{E}\otimes\boldsymbol{E}]}_{\text{coupling}} - \underbrace{\frac{1}{2}\varepsilon_0 J\boldsymbol{C}^{-1}:[\boldsymbol{E}\otimes\boldsymbol{E}]}_{\text{free space}}, \tag{3}$$

with the model parameters μ, λ, c_1, c_2, the right Cauchy-Green tensor $\boldsymbol{C}$, $J = \det(\boldsymbol{F})$ and vacuum permittivity ε_0. From this energy density, the stress and electric displacement field can be derived via

$$\frac{\partial\Omega}{\partial\boldsymbol{F}} = \boldsymbol{P}^{\text{ela}} + \boldsymbol{P}^{\text{ele}} \qquad \frac{\partial\Omega}{\partial\boldsymbol{E}} = -\boldsymbol{D}, \tag{4}$$

respectively [7], while the viscous contribution $\boldsymbol{P}^{\text{vis}}$ might be taken from [6].

1.1 Finite element approach

For numerical treatment, the complex electromechanical coupling equations (1) need to be discretised both in space as well as in time. In this work, at first the spatial discretisation is carried out using hexahedral finite elements and linear shape functions to obtain a system of ordinary differential equations with a spatially discrete configuration. Then, a variational time integration scheme is derived. The variational integrator shows a very good long time energy behaviour [2]. There is neither numerical damping nor an artificial energy gain present and the total energy error is bounded.

Even though the finite element based simulation framework provides a powerful tool to solve electromechanically coupled and dynamic problems of arbitrary geometry, the computational cost is quite demanding. Therefore, so called lumped parameter models are often used for complex tasks like solving optimal control problems.

1.2 Lumped parameter approach

In this work, it is assumed that the deformation of a reference actuator happens symmetrically and volume preserving ($\hat{x}\hat{y}\hat{z} = \hat{X}\hat{Y}\hat{Z}$) as illustrated in Figure 2, such that the deformation field $\boldsymbol{x} = \begin{pmatrix} x_1 & x_2 & x_3 \end{pmatrix}^T$ for a material point $\boldsymbol{X} = \begin{pmatrix} X_1 & X_2 & X_3 \end{pmatrix}^T$ can be written as

$$x_1 = X_1\frac{\hat{x}}{\hat{X}} = X_1\sqrt{\Lambda^{-1}} \qquad x_2 = X_2\frac{\hat{y}}{\hat{Y}} = X_2\sqrt{\Lambda^{-1}} \qquad x_3 = X_3\frac{\hat{z}}{\hat{Z}} = X_3\Lambda, \tag{5}$$

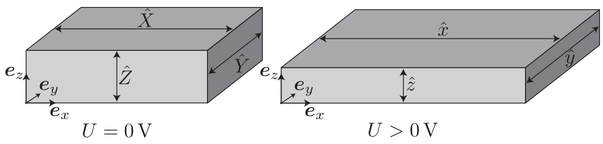

Figure 2. Deformation of DEA cell due to an applied voltage.

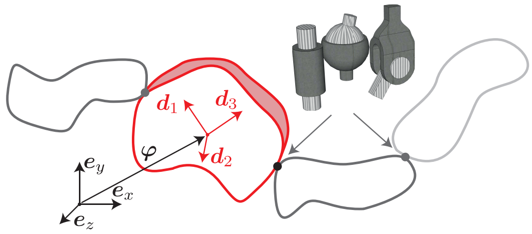

Figure 3. Kinematic chain with director coordinates that span a local Euclidean coordinate system.

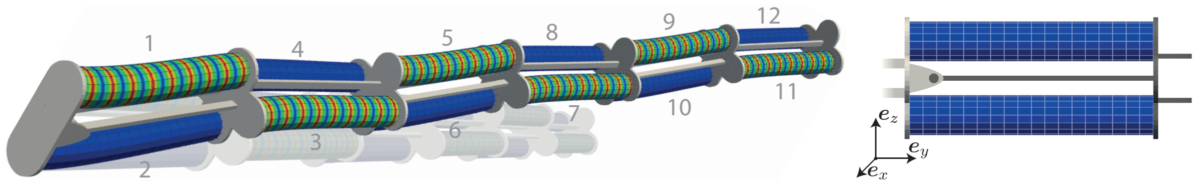

Figure 4. Assembled elephant trunk with 6 rigid bodies and 12 artificial muscles in its initial configuration (light grey) and deflected state (coloured). The colouring refers to the applied electric potential.

with the scalar strain measure $\Lambda = \hat{z}/\hat{Z}$. As a consequence, the deformation gradient $\boldsymbol{F} = \partial \boldsymbol{x}/\partial \boldsymbol{X}$ is spatially constant and the equations of motion (1) can be solved analytically without any further spatial discretisation.

2 Multibody System

To explore the complex behaviour of humanoid robots that are actuated by artificial muscles, the actuator model is coupled with a multibody system. The multibody system consists of a chain of rigid bodies that are connected by joints, as illustrated in Figure 3. The coupling between the finite element muscle model and the rigid bodies is formulated at configuration level, where Lagrange multipliers account for constraint forces, leading to differential algebraic equations of index-3. A well-chosen set of redundant configuration variables for the multibody system [2] avoids rotational degrees of freedom and leads to linear coupling constraints. As a result, the coupling between the artificial muscles and the multibody system can be formulated in a very modular way that allows for easy future extension [5]. The variational integrator allows to solve the index-3 system directly and with numerical accuracy, avoiding index reduction approximations.

3 Optimal Control

When applying constant voltages to the muscles, the actuated system starts oscillating (due to inertia terms) until viscoelastic contributions dissipate the kinetic energy and the system approaches its steady-state. To avoid these oscillations, the direct transcription method DMOCC [3] is applied, leading to an optimisation problem of the form

$$\min_{\underline{x}} \mathcal{J}(\underline{x}) \quad \text{subject to} \quad \underline{c}(\underline{x}) = \underline{0}, \tag{6}$$

where $\underline{x}$ contains the discrete configuration variables and controls for all time steps and the equality constraints $\underline{c}$ are composed of (a) variational time integrator equations to obtain physically meaningful solutions, (b) prescribed initial and final states and (c) path constraints such as control bounds. The objective $\mathcal{J}$ allows to specify a function that is being minimised by the optimised control trajectory.

The example model is a kinematic chain that consists of series-connected revolute joints as illustrated in Figure 4. Each joint is rotated relative to its predecessor by 90 degrees around the

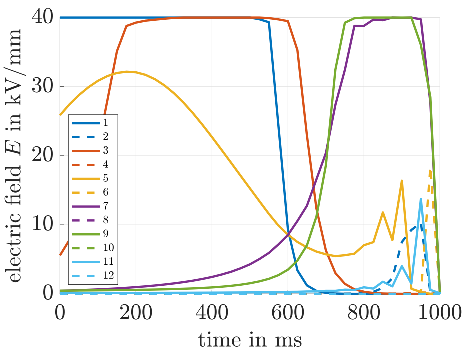

Figure 5. Optimised voltage trajectories in the 12 artificial muscles.

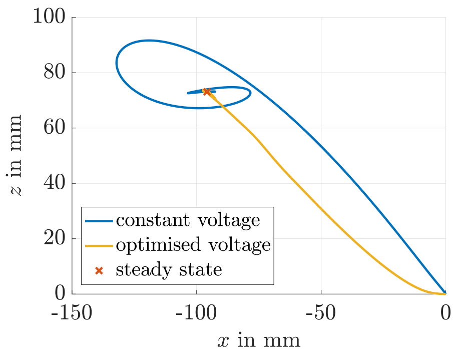

Figure 6. Elephant trunk tip motion path.

y-axis and actuated via two artificial muscles in agonist-antagonist configuration. This setting allows for motion in all space dimensions and is further referred to as elephant trunk. In the present example, the elephant trunk shall move from its initial configuration to the deflected state as depicted in Figure 4 and the objective function to be minimised is the control effort regarding the applied voltages. In Figure 5, the optimised voltage trajectories in all actuators are illustrated. It can be observed that there is a complex interaction between agonists (odd muscles, solid) and some antagonists (even muscles, dashed). In Figure 6, the motion of the elephant trunk tip in the (x-z)-plane can be observed. While the constant voltages lead to overshooting and oscillations around the steady state, the optimised voltage trajectories actuate the system directly towards its final configuration.

Conclusions

The utilised variational time integration scheme turned out to be very suitable for solving electromechanically coupled problems. Apart from the preservation characteristics and the good energy behaviour, the integrator allows to solve algebraic constraints exactly at the discrete time nodes. This allows for a neat coupling between the artificial muscles and the actuated structure. Optimal control theory provides a suitable tool for avoiding oscillations that are inherent with the elastic nature of the actuators and for obtaining optimised voltage control trajectories.

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