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The TPM^2-Method: A two-scale homogenization scheme for fluid saturated porous media

Florian Bartel[1]*, Tim Ricken[2], Jörg Schröder[3] and Joachim Bluhm[3]

Micro Abstract
This contribution will present a two-scale homogenization (FE^2-Method) approach for fluid saturated porous media with a reduced two-phase material model (TPM), which covers the behaviour of large poro-elastic deformation. The main aspects of theoretical derivation for the weak form, the lower level boundary conditions under consideration of the Hill-Mandel homogeneity condition and the averaged macroscopic tangent moduli will be pointed out and a numerical example will be shown.

[1]Chair of Mechanics, Structural Analysis and Dynamics, TU Dortmund University, Dortmund, Germany
[2]Institute of Structural Analysis and Dynamics in Aerospace Engineering, Stuttgart University, Stuttgart, Germany
[3]Institute of Mechanics, University of Duisburg-Essen, Essen, Germany
***Corresponding author**: florian.bartel@tu-dortmund.de

Introduction to TPM^2-Method

The TPM^2-Method is considered as the Theory of Porous Media (TPM), cf. [1] or [2], embedded in the two-scale homogenization environment of the FE^2-Method, see [3], [4]. In this framework we are able to describe large poro-elastic deformation and fluid flow through porous media as well as taking into account the discrete geometry of microscopic structures. Hence, TPM^2-Method allows us to solve two-scale, non-linear, coupled and time dependent problems for bodies with a porous microstructure. As is with the standard TPM formulation, the governing equations - balance of mass, momentum and moment of momentum as well as the entropy inequality - are considered for all phases. The degrees of freedom considered are displacement and hydrostatic pressure, which may be used to determine stresses and volumetric fluid flows. For the micro-macro transition we transfer the macroscopic quantities such as the deformation gradient, fluid stress and pressure gradient to the microstructure and receive an averaged material tangent as a response. Hereby, we satisfy Hill-Mandel homogeneity condition.

1 Continuum mechanical two-scale, two-phase treatment

We consider a general body $\overline{\mathrm{B}}$ consisting of a fluid saturated porous media on the macroscopic scale as shown on the left hand side in Fig. (1). The macroscopic material points for the solid and the fluid phase are defined as $\overline{\mathbf{x}_\mathrm{S}}$ and $\overline{\mathbf{x}_\mathrm{F}}$, respectively. The domain for the Dirichlet boundary condition for the displacements of the solid is given as $\partial\overline{\mathrm{B}_\mathbf{u}}$ and for the pressure of the fluid as $\partial\overline{\mathrm{B}_\lambda}$. On the domain for the Neumann boundary condition we regard external forces on the solid as a normal traction tensor $\bar{\mathbf{t}}$ as well as the normal of the volumetric flow rate $\overline{\mathrm{n}^\mathrm{F}\mathbf{w}_\mathrm{FS}}$ for the fluid. For the solution of the partial differential equations of the macroscopic boundary value problem we make use of the Finite Element Method and receive an approximation for the macroscopic body. Furthermore, the macroscopic material tangent $\overline{\mathbf{A}}$ and the macroscopic Right-Hand-Side $[\overline{\mathbf{P}}, \overline{(\mathbf{E}_\mathrm{S})'_\mathrm{S} \cdot \mathbf{C}_\mathrm{S}\mathrm{J}_\mathrm{S}}, \overline{\mathrm{n}^\mathrm{F}\mathbf{w}_\mathrm{FS}}]^\mathrm{T}$ will be evaluated on each Gaußian point by a volume averaged solution of the underlying microscopic structure named Representative Volume Element

(RVE), which is also approximated with the Finite Element Method in the same way as illustrated in Fig. (1) on the right hand side.

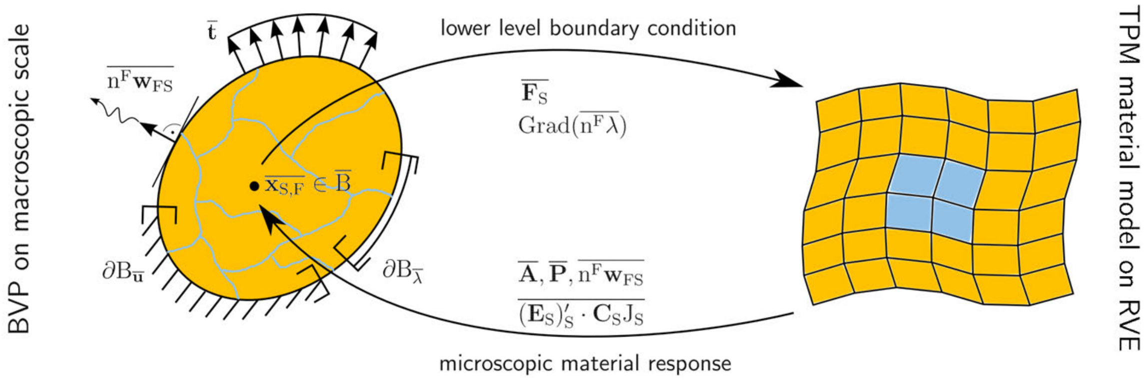

Figure 1. Illustration of a coupled multi-scale theory for fluid saturated porous media.

2 Numerical experiment: Flow through porous body with varying microstructure

As a numerical experiment, we considered a clamped body consisting of a porous material and analyzed the macroscopic deformation behaviour by applying a flow from the left to the right hand side and varied the underlying microsturucture. The macroscopic Boundary Value Problem (BVP) is illustrated in Fig. 2. We regarded a rectangular body, where the displacement degrees of freedom were fixed on the left hand side. The flow through the body was achieved by applying a pressure load ($\lambda = 10\,\mathrm{N/mm^2}$), which was linearly increased over 50 time increments. On the right hand side the pressure was set to zero. The macroscopic body was meshed with 320 finite elements. The evaluation points were on element 10, 290 and 310. We designed three different microstructures, one is isotropic and two are anisotropic. As an example, in Fig. 3 i) one of the anisotropic microstructures is illustrated, from which we extracted the geometry of the RVE (Fig. 3 ii)), which was discretized with 10 × 10 finite elements (Fig. 3 iii)). The black and white domains consist of porous materials each with different material properties. The black material is more stiff and less permeable in relation to the white material. The numerical values for the Lamé constants and Darcy permeability are listed in Tab. 1. As lower level boundary condition, a mixed Dirichlet-Reuss type is chosen.

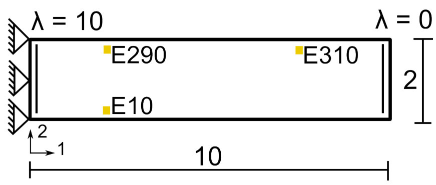

Figure 2. BVP: Flow through porous cantilever.

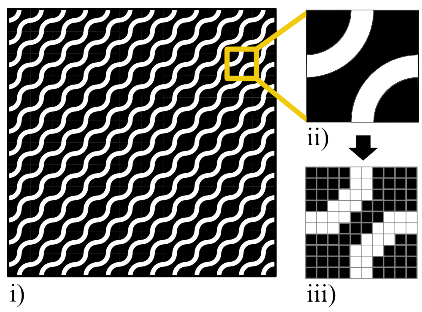

Figure 3. i) Microstructural design, ii) Representative Volume Element (RVE), iii) Discretized RVE with 10×10 finite elements and two different material parameter sets for low and high permeability.

Table 1. Material constants for the different domains of the microstructure

Material constants	**1st Lamé** μ^S	**2nd Lamé** λ^S	**Darcy** k_d
Black domain	26	287	0.1
White domain	16	187	1.0

3 Results of coupled transient multiscale simulation

In Fig. 4 the results of the numerical experiment described in section 2 are shown. We made use of the same BVP but applied three different RVEs. The first RVE (Fig. 4 a), d), g)) contained a hard inclusion and was symmetric about both axes which lead to an isotropic behavior. The second RVE applied in Fig. 4 b), e) and h) contained little cannulas with a higher permeability which lead to an allocated upward flow direction. Hence, the pressure increased in the upper part of the body leading to a greater hydrostatic expansion and a global deformation downwards. The third RVE was identical to the second one however it was rotated by 90 degrees. Here, the fluid flow was forced to the lower part of the macroscopic body which lead to a higher pressure in the lower area and a upward global deformation. The plots of the deformation of the geometry are scaled with a factor 10^2.

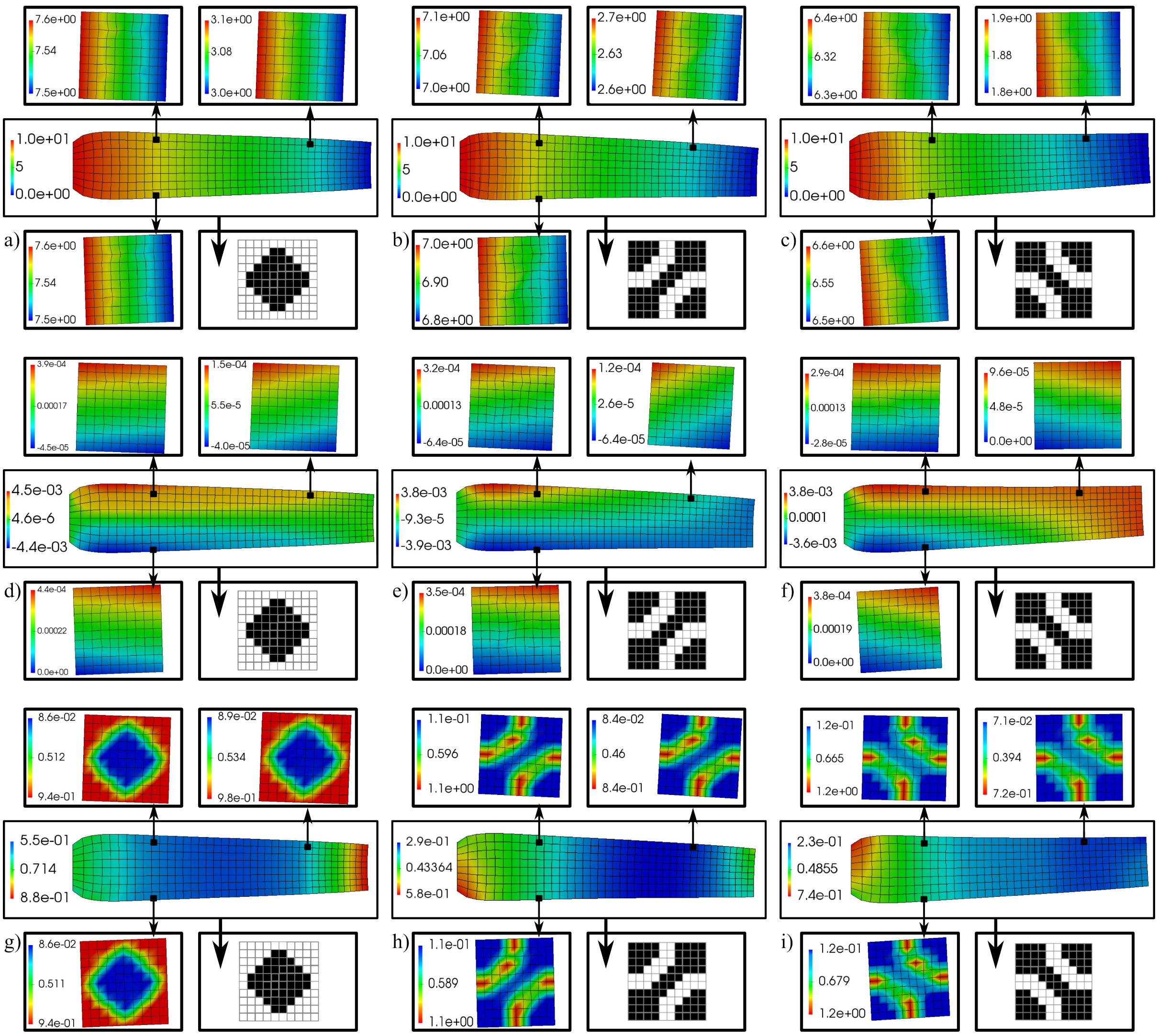

Figure 4. Numerical results on macroscale as well as on microscale for three evaluation points applying three different RVEs. In a), b) and c) the pressure distributions, in d), e) and f) the vertical displacements and in g), h) and i) the horizontal volumetric flows are plotted.

4 Performance study for parallel multi-scale computations

We run numerical examples with the coupled multi-scale approach explained in section 2. In Fig. 5 you can see the results of a compression test which shows a general proof of concept in 3 dimensions. However, the microstructure only consists out of 27 elements due to the high computational runtime. A real problem would need considerably more elements. Hence, we have to improve the performance of the calculation. Fortunately, the FE^2-Method is very applicable for parallel computation. The microstructures can be solved in parallel as indicated in Fig. 6. To verify the improvement of parallel computation for multi-scale problems we investigate in a performance study which you can see in Fig. 7. The calculations run on a high performance cluster called LIDOng at TU Dortmund University with the software FEAP 8.4 from UC Berkeley compiled with Intel Fortran, C++ and MPI on Intel Xeon processors. We analyzed different mesh sizes for the micro- and the macroscale for an increasing number of

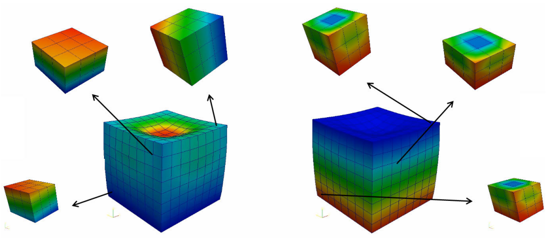

Figure 5. Results for a 3D coupled, two-scale compression test. Displacements (left) and pressure distribution (right).

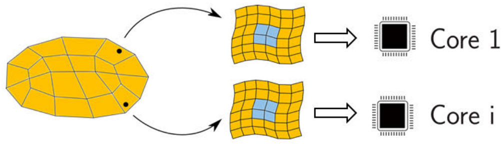

Figure 6. Schematic for parallel multi-scale computation.

processors. The dashed line is the optimum of speed up which can not be achieved, because some parts of the program always run only serial. However, if the relation between micro and macro discretization is chosen properly, there is an appreciable performance improvement.

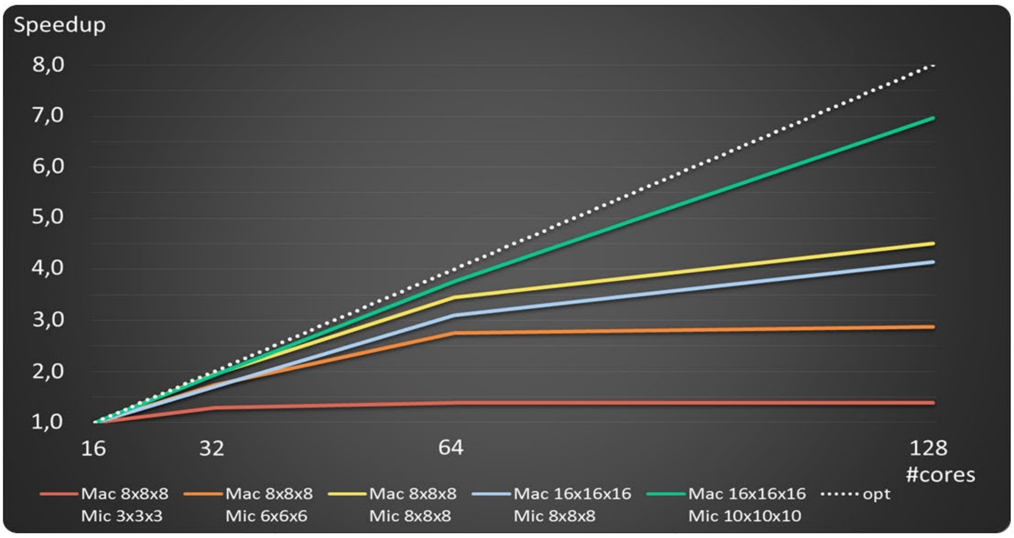

Figure 7. Results of the performance study by increasing the number of cores for different mesh sizes for a multi-scale simulation.

Conclusions

We developed a numerical scheme for solving two-scale, two-phase, non-linear, coupled and time dependent problems for fluid saturated porous materials. The numerical experiment shows the proof of concept and the strong influence of the microstructural design on the macroscopic boundary value problem. As a challenge the high computational effort has to mentioned, therefore, the application of parallel computation strategies is absolutely essential for dealing with realistic problems. Nevertheless, we are convinced, that the TPM^2-Method is a very promising tool for the development and optimization of microstructures, especially in the fields of environmental or biomedical engineering.

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