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On the modelling of evolving material symmetries in finite strain plastic deformations

Tobias Asmanoglo[1]*, Jia Lu[2], Andreas Menzel[1,3] and Panayiotis Papadopoulos[4]

Micro Abstract
Motivated by the experimental findings on sheet metal presented in (J. Mech. Phys. Solids 45, 22, 841–851, 1997) we focus on the elaboration of a specific model which allows us to capture the evolution of the plastic anisotropy that is induced by finite strain plastic deformations. We discuss evolution equations for the structural tensor that characterises the material symmetry group and show that the finite element based simulation results are in good agreement with experimental findings.

[1]Institute of Mechanics, TU Dortmund University, Dortmund, Germany
[2]Center for Computer-Aided Design, University of Iowa, Iowa, United States
[3]Division of Solid Mechanics, Lund University, Lund, Sweden
[4]Department of Mechanical Engineering, University of California, Berkeley, United States
***Corresponding author**: tobias.asmanoglo@tu-dortmund.de

Introduction

The consideration of the plastic anisotropy of a crystalline solid and its evolution when undergoing finite deformations e.g. due to rolling processes plays a crucial role for the simulation of today's engineering applications. In an early contribution, Kim and Yin experimentally analysed the yielding behaviour of cold-rolled sheet metal by means of uni-axial tension tests with the focus of the investigations lying on the evolution of the yield function's symmetry group, see [2]. In particular the experimental findings suggest that the initial orthotropic yielding behaviour of the samples is maintained throughout the deformation process and that a rotation of the yield function's symmetry group is observable. Motivated by these findings Harryson and Ristinmaa [1] proposed a thermodynamically consistent model to capture the experimentally observed evolution of the symmetry group which firmly relies on the multiplicative split of the deformation gradient and hence on the introduction of an intermediate configuration. In this contribution, however, a different approach will be followed in which the intermediate configuration and the multiplicative split of the deformation gradient are only used from the viewpoint of physical motivation, cf. [3].

1 A covariant approach for the simulation of evolving anisotropies

In 2004 Lu and Papadopoulos proposed a theory for the modelling of evolving material symmetries in the context of finite strain plasticity which is based on the postulate of covariance and which can be regarded as an extension to their earlier works in the context of anisotropic elasticity, see e.g. [3]. Since the fundamental theoretical developments presented in [4] will serve as a basis for the specific model to be developed in this contribution, we will briefly summarise the main findings and governing equations.

Assuming the general form of the stored energy function

$$W = \widehat{W}\left(\boldsymbol{F}\cdot\boldsymbol{F}_{\mathrm{p}}^{-1}, \boldsymbol{g}^{\flat}, \overline{\boldsymbol{G}}^{\flat}, \kappa\right) \tag{1}$$

with deformation gradient tensor $\boldsymbol{F}$, the plastic deformation gradient tensor $\boldsymbol{F}_{\mathrm{p}}$, the metric of the intermediate configuration $\overline{\boldsymbol{G}}^{\flat}$, the metric of the spatial configuration $\boldsymbol{g}^{\flat}$ and the scalarvalued internal variable κ, it is shown that invoking the postulate of covariance yields the same constitutive reduction as invariance with respect to superposed rigid body motions on the intermediate and spatial configuration, see also [5,6]. Specifically, we find that the stored energy function (1) can equivalently be expressed in terms of

$$W = \widetilde{W}\left(\boldsymbol{C}, \boldsymbol{C}_{\mathrm{p}}, \kappa\right) \tag{2}$$

with $\boldsymbol{C}$ denoting the right Cauchy-Green tensor and $\boldsymbol{C}_{\mathrm{p}}$ denoting the right Cauchy-Green plastic deformation tensor, which is treated as a primitive variable. However, it can easily be shown by taking into account referential covariance that a material which is characterised by (2) is isotropic. To account for specific material symmetries, structural tensors $\boldsymbol{A}_i$ which characterise the respective material symmetry group may additionally be included in the list of arguments such that the originally anisotropic tensor function may be rewritten as a covariant function in the extended list of arguments according to

$$W = \breve{W}\left(\boldsymbol{C},\, \boldsymbol{C}_{\mathrm{p}}, \boldsymbol{A}_i, \kappa\right) \quad . \tag{3}$$

Motivated by the physics of single crystals and based on the assumption that the plastic deformation leaves the lattice structure unaltered, the material symmetry group is introduced as a subgroup of the Euclidean orthogonal group in the intermediate configuration. Viewed relative to the reference configuration, the symmetry group is no longer Euclidean but found to be a subgroup of the $\boldsymbol{C}_{\mathrm{p}}$-orthogonal group. Since the structural tensors characterise the material symmetry group, the symmetry group's evolution can be modelled in terms of appropriate evolution equations for the structural tensors which need to be consistent with the evolution equation for $\boldsymbol{C}_{\mathrm{p}}$.

To be specific, it has been shown in [4] that the rate equation for the plastic metric is of the form

$$\dot{\boldsymbol{C}}_{\mathrm{p}} = \boldsymbol{C}_{\mathrm{p}} \cdot \boldsymbol{\Gamma}_{\mathrm{p}} + \boldsymbol{\Gamma}_{\mathrm{p}}^{\star} \cdot \boldsymbol{C}_{\mathrm{p}} \quad , \tag{4}$$

with an arbitrary $\boldsymbol{C}_{\mathrm{p}}$-symmetric tensor $\boldsymbol{\Gamma}_{\mathrm{p}}$ that requires constitutive specification. Note, that $\boldsymbol{\Gamma}_{\mathrm{p}}$ and the rate equation (4) may also be motivated based on the multiplicative split of the deformation gradient. In order to be consistent with (4) it was deduced that the rate equations for the structural tensors need to be of the form

$$\dot{\boldsymbol{A}}_i = \underbrace{\left[-\boldsymbol{\Gamma}_{\mathrm{p}} \cdot \boldsymbol{A}_i - \boldsymbol{A}_i \cdot \boldsymbol{\Gamma}_{\mathrm{p}}^{\star}\right]}_{①} + \underbrace{\left[-\boldsymbol{W}_i \cdot \boldsymbol{A}_i - \boldsymbol{A}_i \cdot \boldsymbol{W}_i^{\star}\right]}_{②} \quad . \tag{5}$$

Essentially, (5) implies an additive split of the rate equation into two contributions. The first contribution, termed convected evolution, links the evolution of the structural tensor to the evolution of the plastic deformation tensor $\boldsymbol{C}_{\mathrm{p}}$. The second contribution, however, is independent of the evolution of $\boldsymbol{C}_{\mathrm{p}}$ and hence allows the structural tensor to evolve (to some extent) independently of the plastic deformation. This contribution will be referred to as residual evolution and is specified in terms of the $\boldsymbol{C}_{\mathrm{p}}$-skew tensors $\boldsymbol{W}_i$. It can be shown that the residual-type evolution allows the structural tensor to spin relative to the continuum. At this stage, constitutive functions which are subject to certain restrictions imposed by the second law of thermodynamics need to be furnished for $\boldsymbol{\Gamma}_{\mathrm{p}}$ and $\boldsymbol{W}_i$. In this contribution we will not discuss those restrictions in detail but will rather present some first results of a prototype model which is based on the theoretical foundations summarised in this section.

2 Prototype model

The experimental investigations on cold-rolled sheet metal carried out by Kim and Yin [2] suggest that the symmetry group of the yield function undergoes a rotation such that the principal material axes align with the principal loading directions. Clearly, the symmetry group of the

λ	μ	R_{inf}	ε_0	a_1	a_2	a_3	a_4	a_5	a_6
121.2 GPa	80.7 GPa	0.5	2 MPa	$19.6\,\frac{1}{\text{GPa}^2}$	$21.1\,\frac{1}{\text{GPa}^2}$	$296.4\,\frac{1}{\text{GPa}^2}$	$-21.6\,\frac{1}{\text{GPa}^2}$	$0.0\,\frac{1}{\text{GPa}^2}$	$0.0\,\frac{1}{\text{GPa}^2}$

Table 1. Material parameters used in the finite element simulations.

elastic response may be different from that of the yield function. For the sake of simplicity, we will thus assume an isotropic elastic material response and a yield function of orthotropic-type. In particular, the stored energy function is assumed to be additively composed of an elastic and plastic part, that is

$$\begin{aligned}\breve{W} =& \frac{\mu}{2}\left[\operatorname{tr}\left(\boldsymbol{C}\cdot\boldsymbol{C}_{\text{p}}^{-1}\right) - 2\,\ln\left(\sqrt{\det\left(\boldsymbol{C}\cdot\boldsymbol{C}_{\text{p}}^{-1}\right)}\right) - 3\right] + \frac{\lambda}{2}\,\ln^2\left(\sqrt{\det\left(\boldsymbol{C}\cdot\boldsymbol{C}_{\text{p}}^{-1}\right)}\right)\\ &+R_{\text{inf}}\left[\kappa + \varepsilon_0\exp\left(-\frac{\kappa}{\varepsilon_0}\right)\right] \quad ,\end{aligned} \tag{6}$$

with material parameters λ, μ, R_{inf} and ε_0 according to Table 1. Introducing the referential Mandel-type stress tensor

$$\boldsymbol{M} = 2\,\boldsymbol{C}\cdot\frac{\partial\breve{W}}{\partial\boldsymbol{C}} \tag{7}$$

and denoting its deviator by $\boldsymbol{M}_{\text{dev}}$, a referential representation of an orthotropic yield function in terms of the Mandel stress tensor of the intermediate configuration is given by

$$\begin{aligned}f =& \Big[a_1\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}^2\right) + a_2\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}\cdot\boldsymbol{A}_{\text{p}}\right)\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}\cdot\boldsymbol{A}_{\text{p}}\right) + a_3\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}\cdot\boldsymbol{A}_{\text{p}}^2\right)\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}\cdot\boldsymbol{A}_{\text{p}}^2\right)\\ &+ a_4\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}\cdot\boldsymbol{A}_{\text{p}}\right)\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}\cdot\boldsymbol{A}_{\text{p}}^2\right) + a_5\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}^2\cdot\boldsymbol{A}_{\text{p}}\right) + a_6\operatorname{tr}\left(\boldsymbol{M}_{\text{dev}}^2\cdot\boldsymbol{A}_{\text{p}}^2\right)\Big] - 1 + q\end{aligned} \tag{8}$$

where the abbreviation $\boldsymbol{A}_{\text{p}} = \boldsymbol{C}_{\text{p}}\cdot\boldsymbol{A}$ was used and the energetic dual to κ was introduced as

$$q = -\frac{\partial\breve{W}}{\partial\kappa} \quad . \tag{9}$$

The specific values for the material parameters a_1-a_6 are again provided in Table 1. Note, that the assumption of an associated-type flow rule

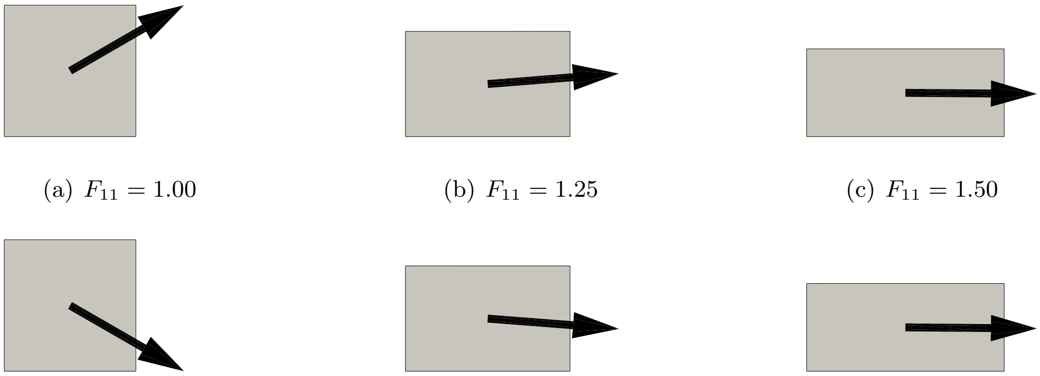
$$\boldsymbol{\Gamma}_{\text{p}} = \lambda_{\text{p}}\,\frac{\partial f}{\partial\boldsymbol{M}} = \lambda_{\text{p}}\,\boldsymbol{\Lambda}_{\text{p}} \quad , \tag{10a}$$

$$\dot{\kappa} = \lambda_{\text{p}}\,\frac{\partial f}{\partial q} \quad , \tag{10b}$$

together with the specific form of the yield function (8) and the stored energy function (6) results in a saturation-type hardening behaviour. With regard to the rate equation (5), the plastic spin tensor $\boldsymbol{W}$ additionally requires constitutive specification. For the finite element simulations to be presented in Section 3 we assume $\boldsymbol{W}$ to be of the form

$$\boldsymbol{W} = \lambda_{\text{p}}\,c_{\boldsymbol{W}}\,\frac{1}{2}\left[\boldsymbol{A}\cdot\boldsymbol{C} - \boldsymbol{C}_{\text{p}}^{-1}\cdot\boldsymbol{C}\cdot\boldsymbol{A}\cdot\boldsymbol{C}_{\text{p}}\right]\,\boldsymbol{A}:\boldsymbol{C} \quad . \tag{11}$$

By construction, the plastic spin tensor (11) is inactive if the deformation is purely elastic, for the plastic multiplier λ_{p} is found to be zero in this case. Moreover, the material parameter $c_{\boldsymbol{W}}$ has been introduced to control the speed of the plastic spin. Further details on the derivation and on the (physical) motivation for the specific form of the spin tensor are, however, beyond the scope of this contribution.



(a) $F_{11} = 1.00$ (b) $F_{11} = 1.25$ (c) $F_{11} = 1.50$

(d) $F_{11} = 1.00$ (e) $F_{11} = 1.25$ (f) $F_{11} = 1.50$

Figure 1. One-element tension tests for an initial angle of $30°$ (a-c), respectively $-30°$ (d-f), enclosed by the principal loading direction and the first axis of orthotropic material symmetry. Depicted is the evolution of the first axis of orthotropic material symmetry at a chosen quadrature point for increasing tensile deformation.

3 One-element tests

Some first finite element simulation results which are based on the prototype model proposed in Section 2 will be presented in this section. To be specific we will focus on one-element tests subject to the assumption of a plane-strain deformation state as shown in Figure 1. The horizontal displacement of the nodes which are located at the left boundary is fixed while the horizontal displacement of the nodes which are located at the right boundary is prescribed. Moreover, homogeneous Dirichlet boundary conditions in vertical direction will be enforced at the boundary nodes at the bottom of the sample. The material parameter $c_{\boldsymbol{W}}$ is chosen to take a value of $c_{\boldsymbol{W}} = 20000\ \text{Pa}^{-1}$, resulting in a significant rotation of the material symmetry group. As shown in Figure 1, a clockwise-rotation is observable in the case of an initial relative orientation of $30°$ between the first principal axis of material symmetry and the principal loading direction. On the contrary, and in good agreement with the experimental findings presented in [2], a counter-clockwise rotation can be observed for an initially enclosed angle of $-30°$, resulting again in an alignment of the first axis of material symmetry with the principal loading direction.

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