**Proceedings of the 7th GACM Colloquium on Computational Mechanics
for Young Scientists from Academia and Industry
October 11-13, 2017 in Stuttgart, Germany**

Simulation of brittle fracture in shells using a phase-field approach and LR B-splines

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Micro Abstract
We present a phase-field approach to model brittle fracture in plates and shells. For structural analysis, the discretization of the geometry is performed using an isogeometric Kirchhoff-Love shell formulation, extended to local refinement with LR B-splines in order to properly resolve the mesh in the cracked regions, improving the accuracy and efficiency of the analysis.

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Introduction

The simulation of fracture of structural members plays an important role in many engineering applications, both at the design step and during inspections of in-service structures. In this contribution we focus on thin parts like plates and shells.
The structural model employs Isogeometric Analysis (IGA), a very promising technique introduced by Huges et al. [3]. This method uses Non Rational B-Splines (NURBS) for the description of the geometry, as CAD commercial programs do, making the approach very favourable because meshing of the model for the structural analysis is not required. The smoothness of the basis functions and the continuity across the element boundaries make the implementation of a rotation-free isogeometric Kirchhoff-Love formulation for shell elements highly favourable [7].
In the last years, the phase-field approach for the description of brittle fracture has been developed. With this method, it is possible to approximate sharp cracks with a continuous field in which the discontinuity is represented by a smeared transition of the parameter between the values referred to broken and unbroken material.
The phase field approach to brittle fracture was recently coupled with the isogeometric rotation-free formulation for plates and shells by Kiendl et al. [6]. In order to properly resolve the phase field in the cracked regions, fine meshes are required. Due to the tensor product properties of NURBS, local refinement of the mesh is not possible in traditional Isogeometric Analysis. In order to improve the efficiency of the analyses in terms of computational costs, we employed local refinement (LR) of the mesh using B-splines as basis functions, as described by Johannessen et al. [5].

1 Modelling and implementation aspects

The model used for the analyses employs a discretization of the geometry using an isogeometric Kirchhoff-Love shell formulation [7] with LR B-splines [5]. For the characterization of brittle fracture, we follow a variational formulation with the phase field discretized by the same basis functions as the geometry, assuming geometrical linearity (small strains and small deformations).

1.1 Phase field formulation of fracture for shells

The evolution of the crack is controlled by the minimization of the functional of the free energy over the body, as defined by Francfort and Marigo [4]:

$$E(\boldsymbol{\varepsilon}, \Gamma) = \int_{\Omega} \psi_e(\boldsymbol{\varepsilon}) d\Omega + \int_{\Omega} \psi_s(s, \nabla s) d\Omega \tag{1}$$

where the first integral expresses the elastic strain energy. The second one computes the fracture energy by approximating and replacing the traditional integration computed over the set of crack surfaces [2]. In the adopted formulation, the fracture energy density $\psi_s(s, \nabla s)$ depends on the phase field parameter s:

$$\psi_s(s, \nabla s) = G_c \left(\frac{1}{4\ell_0}(1-s)^2 + \ell_0 |\nabla s|^2 \right) \tag{2}$$

in which G_c is the fracture toughness of the material. The approximation of the crack topology by the continuous field s is governed by the length scale parameter ℓ_0 that defines the amount of smearing of the phase field around the fracture surface. For $\ell_0 \to 0$, the solution converges to sharp crack topology and Griffith's linear elastic fracture mechanics solution [8]. Other than being a regularization parameter for the model, ℓ_0 depends also on the properties of the considered material, in particular the fracture stress/strain and the fracture toughness [1]. Thus, the choice of the length scale parameter is crucial for the simulation. The mesh of the model has to be sufficiently fine in the crack region in order to resolve ℓ_0, so a reduced size of the elements is often required in this area.
The strain tensor $\boldsymbol{\varepsilon}$ can be split into its tensile and compressive contribution. This is required for a correct characterization of the fracture process, preventing cracking to occur in compression.

$$\boldsymbol{\varepsilon} = \boldsymbol{\varepsilon}^+ + \boldsymbol{\varepsilon}^-. \tag{3}$$

Consequently, also the strain energy density and the stress tensor can be split in the same components. Characterization of the fracture occurs by degradating the tensile terms according to a degradation function:

$$g(s) = (1-\eta)\, s^2 + \eta, \tag{4}$$

where $0 < \eta \ll 1$ for numerical stability of the model in fully cracked situation.
The strong form of the problem is so obtained as momentum and phase field equations:

$$\operatorname{div} \boldsymbol{\sigma} = \mathbf{0}, \tag{5}$$

$$\left(\frac{4\,\ell_0(1-\eta)\psi_e^+}{G_c} + 1 \right) s - 4\,\ell_0^2 \Delta s = 1. \tag{6}$$

Irreversibly of the cracking process is guaranteed by replacing ψ_e^+ whit the so-called history field $\mathcal{H}$, which expresses the maximum of the positive strain energy density over time [8].
In the rotation-free shell formulation a curvilinear coordinate system is used, with θ^1, θ^2 referring to the midsurface, and θ^3 to the thickness direction. At any point of the shell continuum, the two-dimensional strain tensor can be retrieved by:

$$\boldsymbol{\varepsilon}(\theta^3) = \boldsymbol{\varepsilon}^m + \theta^3 \boldsymbol{\kappa}. \tag{7}$$

For determining this quantity, membrane strains $\boldsymbol{\varepsilon}^m$ and curvature changes $\boldsymbol{\kappa}$ are required, which depend only on the midsurface displacement field. The stress tensor ($\boldsymbol{\sigma}(\boldsymbol{\varepsilon})$) and strain energy density ($\psi_e(\boldsymbol{\varepsilon})$) can be obtained from the strain tensor applying linear elasticity, under the assumption of plane stress.
In the adopted shell formulation, the split of the strain energy surface density (strain energy per unit area of the midsurface) is performed by integrating along the thickness:

$$\Psi_e^{\pm} = \int_{-h/2}^{h/2} \psi_e^{\pm}(\theta^3) d\theta^3. \tag{8}$$

The integral is computed numerically and, at every thickness integration point, the strain tensor is decomposed into compressive and tensile components, in order to degradate the positive component according to $g(s)$ [6]. The energy functional from Equation 1 is so computed, integrating over the shell midsurface:

$$E_{\ell_0}(\boldsymbol{\varepsilon}^m, \boldsymbol{\kappa}, s) = \int_A \left(g(s)\, \Psi_e^+(\boldsymbol{\varepsilon}^m, \boldsymbol{\kappa}) + \Psi_e^-(\boldsymbol{\varepsilon}^m, \boldsymbol{\kappa}) + \Psi_s(s, \nabla s) \right) dA. \tag{9}$$

1.2 Local Refinement of B-splines

By the traditional NURBS (Non Uniform Rational B-Splines) discretization, surfaces can be modelled using two parameters (ξ, η), two sets of knot vectors ($\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$ and $\mathcal{H} = \{\eta_1, \eta_2, ..., \eta_{m+q+1}\}$) and a net of control points $\boldsymbol{P}_{i,j}$, as a tensor product of unidimensional B-splines base functions ($N_{i,p}$ and $M_{j,q}$) of degree $p,\ q$:

$$\boldsymbol{S}(\xi, \eta) = \sum_{i=1}^{n} \sum_{j=1}^{m} N_{i,p} M_{j,q} \boldsymbol{P}_{i,j}. \tag{10}$$

The global knot vector net of lines defines the so-called mesh of the geometry. The tensor product property of traditional B-Splines surfaces allows only a global refinement of the geometry by knot insertion. This means that, if a finer mesh is required only in one part of the patch, the refinement will affect all the geometry and the number of elements will increase considerably, causing an high usage of computational resources (Figure 1b).

(a) Initial mesh **(b)** Tensor product refinement **(c)** Truly local refinement

Figure 1. Lack of local refinement of tensor B-splines [5]

Employing the approach of Johannessen et al. [5], we switch to a formulation in which "weighted" B-splines are taken as base functions. The ith weighted B-Spline for surface geometries is defined as the product of two univariate B-splines $B_{p,\Xi_i}(\xi), B_{q,\mathcal{H}_i}(\eta)$ of degree p, q :

$$\boldsymbol{B}^{\gamma}_{\boldsymbol{\Xi}_i} = \gamma_i B_{p,\Xi_i}(\xi) B_{q,\mathcal{H}_i}(\eta). \tag{11}$$

The univariate local knot vectors $\Xi_i, \mathcal{H}_i$ define the local knot vector $\boldsymbol{\Xi}_i$ on which the B-spline has support. The scalar weight γ simply multiplies the B-splines and it is used for maintaining the partition of unity property during the splitting procedure.
After defining the weighted B-splines, that can be considered as the base functions of the geometry, a LR surface object (referring to the midsurface of the analysed shell) can be expressed as the linear combination of weighted B-Splines and control points:

$$\boldsymbol{S}(\xi, \eta) = \sum_{i=1}^{n} \boldsymbol{B}^{\gamma}_{\boldsymbol{\Xi}_i} \boldsymbol{P}_i, \tag{12}$$

in which n is the number of control points, corresponding with the number of B-Splines. It is important to note that, with this definition, there is only one global index i running over the base functions, excluding the global product properties (that are instead maintained only at

level of the single B-spline). This is extremely useful for allowing the local refinement of the mesh.
Without entering into the details of the algorithm, a weighted B-spline can be split into two new base functions, generating each a new couple of control points and weights (in order to maintain partition of unity) that will replace the precedent structure. The splitting procedure is performed inserting a line that completely traverses the local knot vector of the original B-spline, dividing in two halves the crossed elements, in order to generate new local knot vectors. The application of this procedure allows the local refinement of the mesh (as for example in Figure 1c).

2 Numerical example: simply supported plate

Some potentialities of the LR B-splines and the phase field approach for shells are shown in the numerical example below. A square plate (size=2×2 mm, thickness=0.02 mm), simply supported at all the four sides, is subjected to an uniform transversal pressure. Material parameters are: $E = 190 \times 10^3$ N/mm^2, $\nu = 0.29$, $G_c = 0.295$ N/mm and $\ell_0 = 0.02$mm. The crack is expected to initiate in the plate centre and than to branch and propagate towards the corners, so the local refinement of the mesh was performed in these regions. Full span strategy for local refinement was used (meaning that, once an element is selected for being split, all the base functions supported by the element are split), in order to obtain a larger footprint of the refinement and a more regular grid.
The simulation was run with arc-length control method until failure. For solving the coupled problem, we employ a staggered approach in which, for every time step, the weak forms of the phase field and the momentum equation are solved.

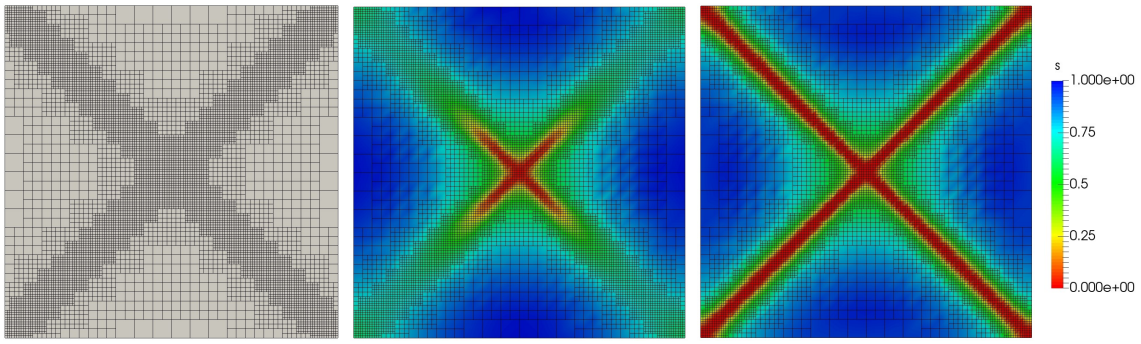

Figure 2. Plate model: locally refined mesh and phase field evolution

Figure 2 shows the LR B-splines mesh and the crack evolution by the phase field, where $s = 1$ indicates undamaged and null value refers to fully cracked material. For the complete model, 8640 elements are used and the minimum size in the refined region is 0.0139 mm. The same numerical test was previously run with a uniformly refined NURBS grid that consisted of 7225 elements, for modelling one quarter of the plate, and a constant element size of 0.0117 mm [6]. The load-displacement curve obtained using the LR B-splines mesh is shown in Figure 3. The pressure required for the failure of the plate is in agreement with the results reported in [6].

Conclusions

A phase-field approach for modelling brittle fracture in plates and shells, discretized by using an isogeometric Kirchhoff-Love element formulation, was presented. The implementation of LR B-splines is a tool that allows the refinement of the mesh in the cracked regions, improving the accuracy and efficiency of the analysis by reducing the total number of elements needed. Good agreement was found with results already obtained using NURBS on equivalent experiment.

Further work will include an extension of the model towards adaptive local refinement of the mesh, in order to be able to describe crack patterns not known in advance.

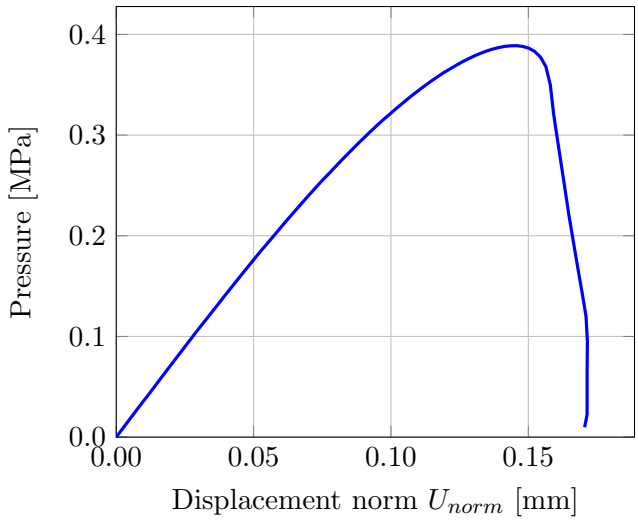

Figure 3. Plate model: load-displacement curve

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