

A Non-Intrusive Global-Local Approach with Application to Phase-Field Modeling of Brittle Fracture

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Micro Abstract

The variational multiscale (VMS) method by Hughes et al. [1] is a well-established framework for the analysis of nonlinear heterogeneous materials and is capable of tackling strain localization in the multiscale framework. In this contribution, we propose a non-intrusive setting of the VMS approach to be applied to the phase-field formulation of fracture [2-5]. The proposed two-scale procedure yields results comparable to the single-scale solution, yet they are obtained with much superior efficiency.

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Introduction

The variational approach to fracture by Francfort and Marigo and the related regularized formulation of Bourdin et al. [2], which is also commonly referred to as phase-field model of fracture, see e.g. the review paper [3], is a widely accepted framework for modeling and computing fracture failure phenomena in elastic solids. The formulation is non-linear and also calls for the resolution of small length scales. Its single-scale treatment is nowadays well-established and known to be computationally demanding [3–5]. Especially for large structures featuring fracture phenomena only in regions of limited extent, the idea of a multiscale approach that enables to treat a localized non-linearity at a lower (local) scale, while dealing with a purely linear problem at an upper (global) one, already introduced for elastoplasticity and fracture problems [6, 8, 13], seems particularly appealing. In view of practical applications in the industrial setting, it is additionally useful to formulate the approach in a non-intrusive fashion to enable the use of existing finite element codes [8, 9, 13].

In this work we aim at developing and efficiently combining the non-intrusive global-local approach with phase-field modeling of brittle fracture. To this end, we equip the formulation with the following features:

- various relaxation techniques are introduced that enable to avoid an overly stiff local response in the case of Dirichlet boundary conditions used at the interface between the two scales [8];
- as an alternative solution to the same problem, generalized Robin-type boundary conditions [9, 10] are imposed at the interface;
- we allow for a non-matching finite element discretization at the interface using a dual mortar method [11] or localized Lagrange multipliers [12].

The proposed two-scale procedure yields results identical to the single-scale solution, yet they are obtained with superior efficiency. We note that the presented method is (algorithmically)

perfectly suitable for parallel computing thus promising further reduction of computational effort.

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