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Contact, Fluid Structure Interaction and Variational Transfer

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Micro Abstract
We present a new and completely parallel approach for fluid-structure interaction, which includes also contact between the elastic structures. Our approach is inspired by the fictitious domain method and makes intensive use of variational transfer between the solid and the fluid and on the possibly contacting surfaces between solids. We present the discretization, the setup of the variational transfer, and efficient methods for the solution of the arising non-linear problems. We present 2D and 3D benchmarks, and a tricuspid heart valve.

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Introduction

In this work we present a fluid-structure interaction (FSI) framework based on the *fictitious domain method* [1] by using a parallel L^2-projection [6] for coupling the two problems and transferring data between the fluid and the solid meshes. Our final goal is simulating the behavior of the heart valve during the entire heartbeat, which requires solving the contact arising between the three leaflets during the closure phase.

1 Method

The main idea of the *fictitious domain method* is modeling the solid phase as immersed in a background fluid phase. The fluid is described in an Eulerian fashion (fixed mesh), while we use Lagrangian meshes for the solid structure. The coupling between the two phases is achieved with overlapping domain decomposition. Specifically, our coupling is constructed by means of L^2-projections where a Lagrange multiplier is used to weakly enforce the velocity vector constraint along the interface-boundary between the solid and the fluid.

For solving the arising nonlinear-system of discrete equations we adopt a staggered approach within a fixed-point iteration, hence the fluid and the solid problems are solved separately with the following four steps: First, we transfer velocities from the solid mesh to the fluid mesh. Second, we solve the fluid dynamics problem by imposing the velocity constraint in the overlap. Third, we compute and transfer the reaction force from the fluid mesh to the boundary of the solid mesh. At last, we solve the solid mechanics problem by imposing the reaction force on the solid (Neumann) boundary.

1.1 Mortar projection

Let $\boldsymbol{V}_s^h = \boldsymbol{V}_s^h(\mathcal{T}_s^h)$ (mortar) the finite element space associated with the solid mesh $\mathcal{T}_s^h$, and $\boldsymbol{V}_f^h = \boldsymbol{V}_f^h(\mathcal{T}_f^h)$ (non-mortar) be the finite element space associated with the fluid mesh $\mathcal{T}_f^h$. In the FSI context we use $\boldsymbol{V}_s^h$ and $\boldsymbol{V}_f^h$ for describing the velocities in the respective phases. For the introduction of the projection operator we define a suitable discrete space of Lagrange multipliers $\boldsymbol{M}_{FSI}^h$. Here the Lagrange multiplier space is associated with fluid mesh, in fact we choose $\boldsymbol{M}_{FSI}^h = \boldsymbol{V}_f^h$.

Now we proceed with the definition of the projection operator $\mathcal{P}\colon \boldsymbol{V}_s^h \to \boldsymbol{V}_f^h$. For a function $\boldsymbol{v}^h \in \boldsymbol{V}_s^h$ we want to find $\boldsymbol{w}^h = \mathcal{P}(\boldsymbol{v}^h) \in \boldsymbol{V}_f^h$ such that following weak-equality condition holds:

$$\int_{I^h} (\boldsymbol{v}_s^h - \mathcal{P}(\boldsymbol{v}_s^h)) \cdot \boldsymbol{\lambda}_{FSI}^h \, dV = \int_{I^h} (\boldsymbol{v}_s^h - \boldsymbol{w}_f^h) \cdot \boldsymbol{\lambda}_{FSI}^h \, dV = 0 \qquad \forall \boldsymbol{\lambda}_{FSI}^h \in M_{FSI}^h, \tag{1}$$

where $I^h = \mathcal{T}_s^h \cap \mathcal{T}_f^h$ is the intersection between the solid mesh and fluid mesh. We express $\boldsymbol{v}_s^h$, $\boldsymbol{w}_f^h$ and $\boldsymbol{\lambda}_{FSI}^h$ in terms of their basis functions, i.e., $\boldsymbol{v}_s^h = \sum_{l \in J_s} v_s^l \boldsymbol{\varphi}_s^l$, $\boldsymbol{w}_f^h = \sum_{j \in J_f} w_f^j \boldsymbol{\theta}_f^j$ and $\boldsymbol{\lambda}_{FSI}^h = \sum_{k \in J_{FSI}} \lambda_{FSI}^k \boldsymbol{\psi}_{FSI}^k$, where J_s, J_f and J_{FSI} are the index sets associated with the nodes of the respective meshes. Hence, we get the so called mortar integrals: $B_{k,l} = \int_{I^h} \boldsymbol{\varphi}_s^l \cdot \boldsymbol{\psi}_{FSI}^k \, dV$, and $D_{k,j} = \int_{I^h} \boldsymbol{\theta}_f^j \cdot \boldsymbol{\psi}_{FSI}^k \, dV$, which can be rewritten in the following algebraic form:

$$\boldsymbol{v}_f = \boldsymbol{D}^{-1} \boldsymbol{B} \boldsymbol{v}_s = \boldsymbol{T} \boldsymbol{v}_s.$$

The transpose of $\boldsymbol{T}$ is used to transfer the reaction force from the fluid to the solid grid.

For discretizing the contact problem between solids we adopt a similar approach based on the L^2-projection [2], where, instead of enforcing the weak-equality condition (1) in the volume, we enforce a weak-inequality condition on the predicted contact surface. The discretization of the Lagrange multiplier is done by means of the biorthogonal basis [7] which allows us to have a diagonal operator $\boldsymbol{D}$, which implies a cheap computation of the matrix $\boldsymbol{T}$.

In the distributed memory of super-computers meshes are arbitrarily distributed and generally unrelated. In [6] we make use of parallel bounding-volume-hierarchies and space-filling curves. This technique allows us to find the intersections and automatically balance the computation of the transfer operator $\boldsymbol{T}$, for both volume and contact without prior knowledge on the distribution of the meshes.

1.2 Contact solution strategy

The algebraic system resulting from the contact problem is solved by adopting a semi-smooth Newton method for the spatial discretization which allows us to treat the inequality constraints arising from the contact problem. In transient settings, for resolving the non-smooth effects caused by the non-penetration and the persistency condition, a suitable stabilized Newmark scheme [5] is required.

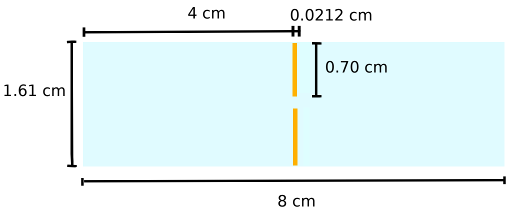
2 Numerical Results

The results are obtained by combining the parallel Finite Element library MOOSE[1] with Utopia[2]. The library MOONoLith[3] is used for identifying the overlapping regions between the fluid and the solid mesh and for coupling the fluid-dynamics and the solid-mechanics problems, and for identifying the contact boundary between solids.

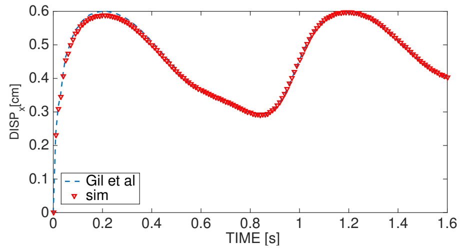
2.1 Validation of the FSI and contact code

We validated the FSI framework by using the benchmark proposed by Gil [3] which consists of two flapping beams embedded into a two-dimensional fluid channel. The fluid is supposed to be Newtonian, while an incompressible Neo-Hookean material is used for the two solid membranes which are characterized by different stiffnesses. A pulsatile non reversible flow is applied as a Poiseuille flow at the inlet (Figure 1a), whereas a non-slip condition is employed on both the bottom and the top of the fluid channel. By analyzing the displacement field of a point placed on the top beam, we get results in very good agreement with those obtained from Gil (Figure 1b).

[1] `http://mooseframework.org`
[2] `https://bitbucket.org/zulianp/utopia`
[3] `https://bitbucket.org/zulianp/par_moonolith`

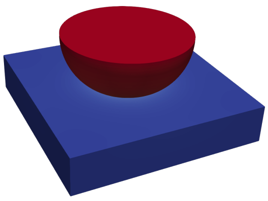


(a) Set-up.

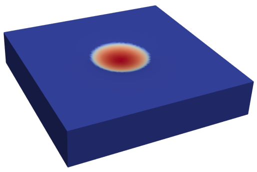


(b) Result.

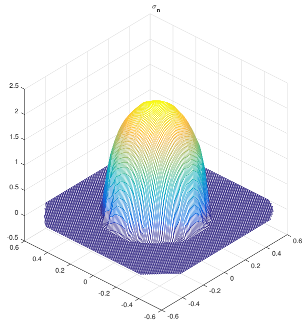
Figure 1. Fluid-structure interaction benchmark.



(a) Quasi-Hertzian contact.



(b) Result with normal stresses.



(c) Normal stresses at contact boundary.

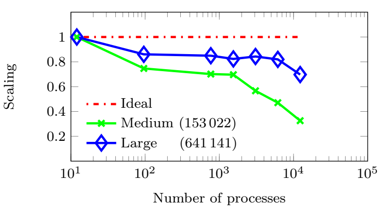
Figure 2. Benchmark for contact between solids.

We validated our contact framework, based on the surface mortar-projection, by solving a quasi-Hertzian problem (Figure 2). By applying corresponding Dirichlet conditions on the top of the semi-sphere, we push the semi-sphere onto the lower plane, where we observe that the normal stresses at the contact boundary correspond to the analytical solution [4].

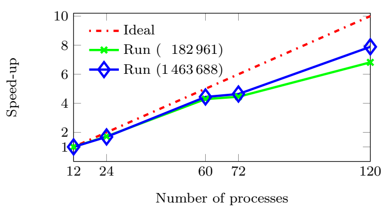
2.2 Scaling and examples

Figure 3 shows weak and strong scaling studies [6] for the assembly of the transfer operator. We run our experiments on the Piz-Daint supercomputer at the Swiss Supercomputing Center (CSCS) utilizing up to 12288 cores. The experiments, with different output sizes for the volume projection, show proper scaling behavior even when the problem is subject to imbalance due to the output-sensitivity arising from the computation of intersections.

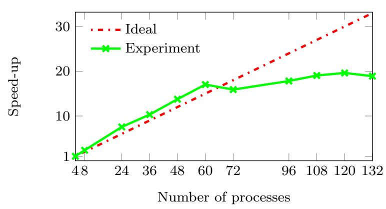
Figure 4 shows the results of several simulations ranging from multi-body contact problems to an heart-valve interacting with the blood-stream.



(a) Weak-scaling (volume).

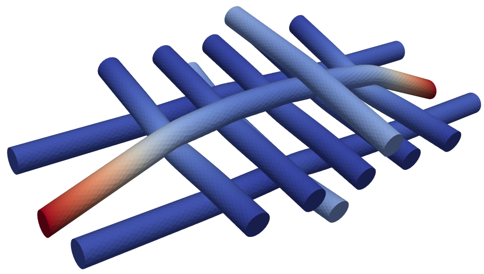


(b) Strong-scaling (volume).

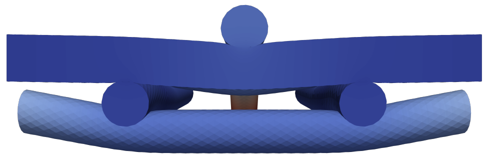


(c) Strong-scaling (surface).

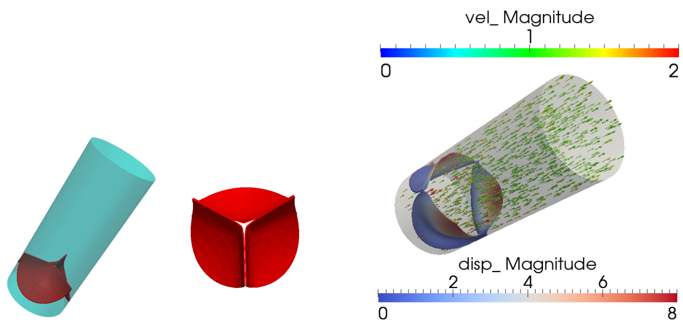
Figure 3. Scaling studies for the parallel volume projection we employ for the fluid-structure coupling (a and b) and for the contact (c).



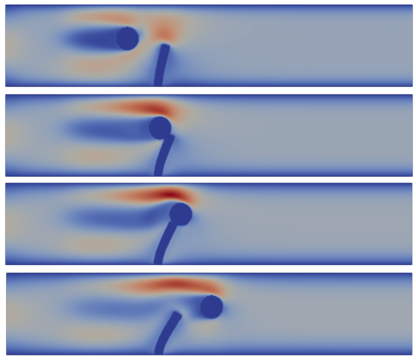
(a) Multi-body contact.



(b) Cut on a three-point contact region.



(c) An heart-valve interacting with the fluid.



(d) Contact between solids inside the fluid.

Figure 4. Example simulations.

Conclusions

In this work developed a framework for the FSI problem including contact between solids. The framework is first validated within the different components by first adopting the FSI benchmark proposed by Gil, then by replicating the normal stresses at the contact boundary of Hertzian contact, and finally by showing the parallel performance of the assembly of the transfer operator.

Acknowledgements

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[5] `http://sccer-furies.epfl.ch`

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