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Eigenerosion Approach for Drucker-Prager Plasticity

Aurel Qinami[1] and Michael Kaliske[1]*

Micro Abstract
The eigenfracture scheme is a suitable technique to model brittle fracture. This method faces challenges when dealing with inelasticity. The contribution at hand is focused on an eigenerosion formulation for Drucker-Prager plasticity. Its binary approach, leading to element stiffness degradation, can be implemented in a straightforward manner into a finite element code. Special attention is paid to the distinction of tension and compression. The method is validated by a numerical example.

[1]Institute for Structural Analysis, Technische Universität Dresden, Dresden, Germany
***Corresponding author**: michael.kaliske@tu-dresden.de

Introduction

Numerical treatment of fracture in linear elasticity is a well established field in computational mechanics. The variational formulation of fracture given in [1] is a solid foundation which provides useful theoretical tools to further develop the description of fracture phenomena. The eigenfracture scheme is a suitable approach to model fracture. Initially presented in [3] in a more applicable form, the method is consolidated in further publications of the above mentioned author.

Nevertheless, dealing with inelasticity in fracture is still a challenge in the scientific community. In particular, for the eigenfracture scheme, the difficulty lies in the energetic treatment of the phenomenon where assigning a specific part of the energy to a specific physical process is not very straightforward. In this work, inelasticity is taken into account by Drucker-Prager plasticity material behavior, which is put into the microplane framework for small strain regime. For the eigenfracture scheme, the version presented in [4] with a volumetric-deviatoric split of the strain tensor is employed. Moreover, the energy contributing to fracture will also depend on the evolution of plasticity in the material.

Drucker-Prager plasticity in the microplane framework

The microplane approach is a very useful model to describe concrete characteristics, which are characterized by a nonlinear and quasi-brittle behavior. As a heterogeneous material, its properties change from isotropic to anisotropic if the material is in its inelastic regime. The idea behind the microplane approach is that the macroscopic strain tensor is projected onto each microplane, where the constitutive relations are defined. At the end, a homogenization takes place to transfer back all the microplane quantities to macroscopic quantities. These microplanes are built around a sphere, which represents a Gauss integration point. The used microplane approach is the one based on a kinematic constraint, where the stress tensor is derived from the strain tensor. The strain tensor is decomposed into a volumetric and a deviatoric part using two projection tensors

$$\epsilon_V = \boldsymbol{V} : \boldsymbol{\epsilon}, \qquad \boldsymbol{\epsilon}_D = \boldsymbol{Dev} : \boldsymbol{\epsilon}\,, \tag{1}$$

where $\boldsymbol{V}$ and $\boldsymbol{Dev}$ are the volumetric and deviatoric projection tensors.

Moreover, with Ψ^{mac} being the macroscopic free energy function and Ψ^{mic} being the microscopic one, the integration over the surface of the sphere is given as

$$\Psi^{mac} = \frac{3}{4\pi} \int\limits_{\Omega} \Psi^{mic} d\Omega \quad . \tag{2}$$

In [5], it is stated that the above integration is accurate if it is calculated with 42 microplanes, and due to symmetry only 21 are sufficent leading to

$$\frac{3}{4\pi} \int_{\Omega} (\bullet)\, d\Omega = \sum_{mic=1}^{21} (\bullet)\, w^{mic} \,, \tag{3}$$

where w^{mic} is the weight of each microplane. More details can be found in [5].

In order to consider inelasticity, the Drucker-Prager plasticity is used to describe the material behavior. This constitutive model is implemented into the microplane framework [6]. The microscopic free energy function is given as

$$\Psi^{mic} = \frac{1}{2} K^{mic} \left(\epsilon_V - \epsilon_V^{pl}\right)^2 + G^{mic} \left(\boldsymbol{\epsilon}_D - \boldsymbol{\epsilon}_D^{pl}\right) \cdot \left(\boldsymbol{\epsilon}_D - \boldsymbol{\epsilon}_D^{pl}\right) + \frac{1}{2} H \kappa^{mic} \quad , \tag{4}$$

where the plastic part of the strains are introduced with the superscript 'pl'. The elastic material parameters are $K^{mic} = 3K$ and $G^{mic} = G$ with K and G being the bulk and shear modulus. H is the hardening stiffness and κ^{mic} the hardening variable. The stresses are derived as

$$\sigma_V = \frac{\partial \Psi^{mic}}{\partial \epsilon_V} = K^{mic} \left(\epsilon_V - \epsilon_V^{pl}\right) \qquad , \qquad \boldsymbol{\sigma}_D = \frac{\partial \Psi^{mic}}{\partial \boldsymbol{\epsilon}_D} = 2G^{mic} \left(\boldsymbol{\epsilon}_D - \boldsymbol{\epsilon}_D^{pl}\right) \quad . \tag{5}$$

For the Drucker-Prager plasticity, the yielding function writes

$$f_{tr}^{mic} = \sqrt{\frac{3}{2} \boldsymbol{\sigma}_D^{tr} \cdot \boldsymbol{\sigma}_D^{tr}} + \alpha \sigma_V^{tr} - \left(\sigma_0 + H \kappa_{n-1}^{mic}\right) \quad , \tag{6}$$

where index 'tr' refers to the trial state. The return mapping algorithm as well as the evolution laws for the internal variables are given in [6]. Moreover, in [6], κ^{hom} is introduced as a measure of the total plastic deformation and it is calculated as

$$\kappa^{hom} = \frac{\frac{3}{4\pi} \int_{\Omega} \kappa^{mic} d\Omega}{\frac{3}{4\pi} \int_{\Omega} d\Omega} \quad . \tag{7}$$

Eigenerosion approach

From the fracture mechanics perspective, the failure mechanism is based on the eigenerosion approach introduced in [3]. There, all elements are characterized by a binary approach, meaning that there can either be intact and still bear load or be failed and their stiffness is reduced to zero. This is described by the parameter d characterizing each element as

$$\begin{cases} d = 0 : & fracture \quad , \\ d = 1 : & no\, fracture \quad . \end{cases} \tag{8}$$

Having this in mind, the stresses for each integration point write

$$\boldsymbol{\sigma} = \frac{3}{4\pi} \int_\Omega \left[\boldsymbol{V} \sigma_V + d\, \boldsymbol{Dev}^T \cdot \boldsymbol{\sigma}_D \right] d\Omega = \sum_{mic=1}^{21} \left[\boldsymbol{V} \sigma_V + d\, \boldsymbol{Dev}^T \cdot \boldsymbol{\sigma}_D \right] w^{mic} \,, \tag{9}$$

where it can be seen that only the deviatoric part of the stress tensor is eroded. The consistent elastoplastic tanget for different cases is given in [6] and is derived from Equation (9) as

$$\frac{\partial \boldsymbol{\sigma}}{\partial \boldsymbol{\epsilon}} = \frac{3}{4\pi} \int_\Omega \left[d c_{dev} + c_{vol} \right] d\Omega \quad , \tag{10}$$

with c_{dev} and c_{dev} being fourth order tensors reresenting the deviatoric and volumetric part of the elastoplastic tangent. From Equation (10), it can be observed that the erosion is applied only to the deviatoric part. In contrary to [4], where the split of the strain tensor follows a spectral decomposition, which makes it able to define tensile and compressive behavior, here a volumetric-deviatoric split takes place.

Finally, the fracture evolution will depend on the comparison between the fracture energy $G_c \Delta A_K$ and the corresponding element energy ΔE_K. If the later one exceeds the former one, the current element is eroded. The above mentioned ΔA_K is the regularized volume of the neighborhood of the current element K, see [3] for more details. Determining ΔE_K is the key to a realistic modeling. In this work, the energy corresponding to the element K is integrated over all Gauss points

$$\Delta E_K = \int_\Omega \Psi^{mac} d\Omega \quad , \quad \text{with} \quad \Psi^{mac} = \left(\Psi^{mac} \right)^{1+p} \quad , \tag{11}$$

where $p = \frac{\kappa^{hom}}{\kappa^{hom}_{crit}} \leq 1$. κ^{hom}_{crit} is a user defined critical value measuring the total plastic deformation.

Numerical example

A four-point bending test of a plane concrete beam with a notch is chosen to validate the approach. The procedure to conduct this experiment is well documented in [2]. Geometry and boundary conditions are given in Figure 1. Moreover the depth of the specimen is 50 mm. The material parameters are $E = 38$ GPa, $\nu = 0.18$, $G_c = 0.13$ N/m, $\sigma_0 = 1.6$ MPa, $\alpha = 0.5$, $H = 0.8$ GPa, $\kappa^{hom}_{crit} = 0.1$.

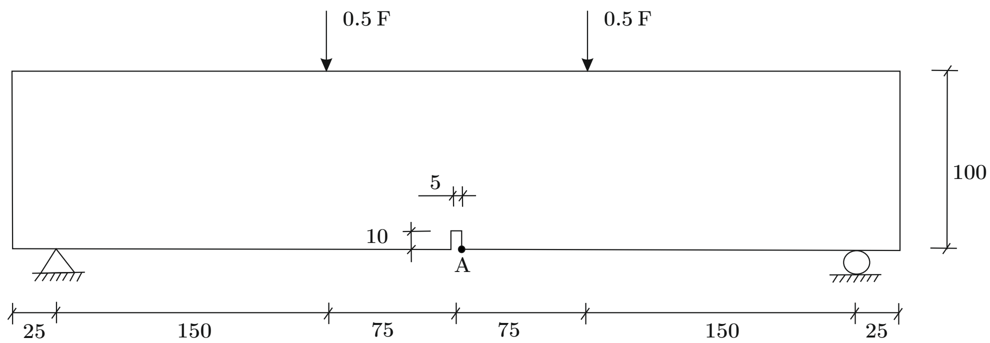

Figure 1. Four-point bending test geometry and boundary conditions

The discretization is done using 1592 hexahedral elements, where smaller element sizes surround the notch. The comparison of load-displacement curves for simulation and experiment is given in Figure 2 a), which shows the relation between reaction forces and deflection of point A. The eigenerosion evolution through the specimen is zoomed in Figure 2 b).

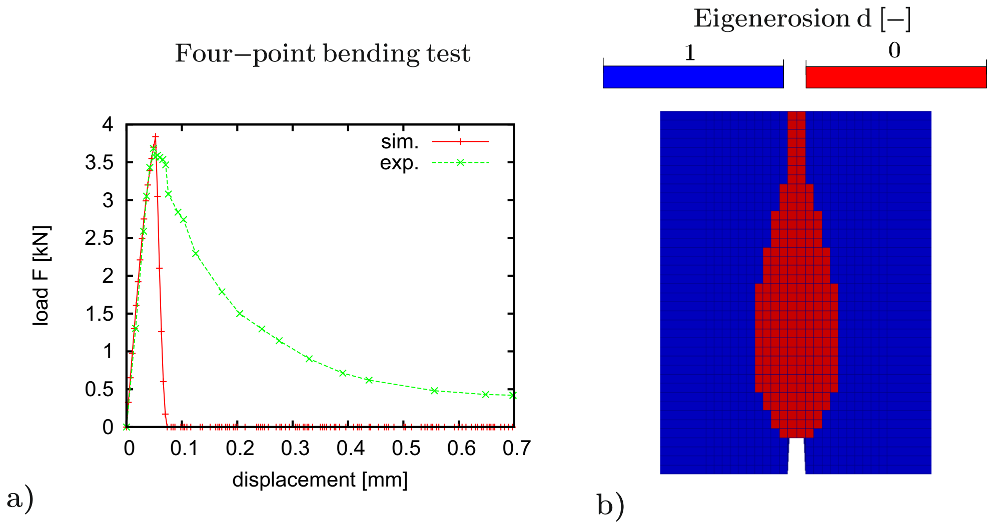

Figure 2. a) Load-displacement curves; b) evolution of eigenerosion around the notch

Two main aspects need to be pointed out from the results. The first one is a quasi brittle behavior that can be seen in Figure 2 a). This makes it difficult to model the same softening behavior described by the experiment. Second, the dependency of energy on the parameter p affects the results in the form of the crack shape given in Figure 2 b). This shape is same as the shape of the evolution of plastic strains in the specimen. Choosing different values for κ_{crit}^{hom} would change the crack shape making it slimmer or wider.

Conclusions

A ductile fracture formulation for the eigenerosion approach is presented in this work. The method is validated in a four-point bending test and the results show good agreement with the experimental ones.

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