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Contact along virtual interfaces: coupling the X-FEM with the mortar discretization

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Micro Abstract
We suggest a computational framework combining the extended finite element method (X-FEM) with the mortar integration for domain-tying and contact problems. We formulate interface conditions between a finite element surface and a virtual surface, passing at any location of another FE mesh (level set). The integration of the internal work employs the X-FEM. The tying or contact constraints are satisfied in the mortar sense. The aimed applications include a structural zoom and wear simulation.

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Introduction

Interface phenomena like contact, friction and wear are complex both with regard to their mathematical description and numerical treatment. Their localized nature lays a strong emphasis on the interface discretization scheme. Ensuring stability and appropriate patch-test performance of these schemes are necessary ingredients to the overall accuracy and robustness of the contact treatment. A relative motion between contacting bodies can lead to material removal (wear) on rubbing surfaces, affecting the contact pressure and thus leading to alteration of the system's global response. Numerical simulation of wear usually involve (1) constitutive local wear laws determining the wear depth evolution at every effective cycle, (2) re-meshing procedures to capture the shape changes at the interface, and (3) field remapping in case of material behavior involving internal variables.

Face-to-face discretization techniques combined with penalty-based or Lagrange-multiplier based treatment of contact/friction constraints form the state of the art methods enabling to handle contact interaction along non-conformal interfaces in a robust way and ensure the accuracy of surface tractions [3]. Mortar and Nitsche methods are the members of this family. The extended finite element method (X-FEM) presents a different technique to handle intra-mesh discontinuities: shock waves, oxidation fronts, composite materials, voids and cracks. Combining the face-to-face contact formulation with the X-FEM method presents an attractive option to treat contact problems along virtual surfaces/interfaces with incompatible meshes: the face-to-face discretization ensures an accurate treatment of contact and the X-FEM ensures independence of the finite element mesh. The virtual contact surface embedded in the volumetric mesh can incorporate geometrical aspects, such as roughness, and can evolve in time due to wear and/or third body accumulation. Here we briefly outline the main aspects of such a coupling between the mortar and X-FEM methods, and present a few examples.

Computational approach

In this work, features of mortar methods in the context of contact [4,5], and of the X-FEM in the context of void/inclusion [2] treatment are exploited to formulate a generalized framework in 2D. As illustrated in Fig. 1, we propose to use the virtual interface Γ_v, to describe the geometrical changes resulting from wear. The virtual interface Γ_v acts as a slave/master surface of domain Ω_1 getting in contact with the contact surface of domain Ω_2. Surface Γ_v coincides with the unworn surface $\Gamma_{c1} \subset \partial\Omega_1$ at the beginning of the simulation. Upon loading, the material of Ω_1 is worn-out and the surface Γ_v propagates into the bulk to capture the surface evolution. This approach includes: (i) the level set method (LSM) to describe the location of worn-surface, (ii) X-FEM Heaviside enrichment (topological) within elements intersected by the worn-surface, and (iii) the mortar method to model contact along the interface made of Γ_v and Γ_{c2}. We use a monolithic augmented-Lagrangian formulation [1] to resolve the contact inequality constraints for both the normal and tangential directions.

The mortar interface virtual work (δW^{mortar}), is evaluated by setting the virtual boundary Γ_v as the non-mortar side and Γ_{c2} as the mortar side. The gap function $g(x, \tilde{x})$ determines here the integral gap measure, which depends on actualized positions of the boundary Γ_{c2} and of the nodal positions of the blending elements intersected by Γ_v. Tilded notations denote quantities of the domain Ω_1 subject to wear. The contribution of the domain Ω_1 (δW_1^{int}) to the global virtual work of the system is evaluated using the X-FEM enrichment (1). The worn out volume Ω_w evolves with time and is evaluated based on the sliding distance and the local contact pressure. As illustrated in Fig. 1, the body subject to wear is modelled using the X-FEM framework, i.e. the internal virtual work is integrated in a classical way in elements non-intersected by the virtual boundary and only partially integrated on the unworn volume of the blending elements.

$$\delta W_1^{int} = \int\limits_{\tilde{\Omega}=\Omega_1 \setminus \Omega_w} \tilde{\boldsymbol{\sigma}} : \delta\tilde{\boldsymbol{\varepsilon}} \, d\tilde{\Omega}, \quad \delta W^{mortar} = \int\limits_{\Gamma_{c2}} \delta \begin{cases} \left[\lambda(x)\, g(x,\tilde{x}) + \dfrac{\varepsilon}{2} g^2(x,\tilde{x}) \right] d\Gamma, & \lambda + \varepsilon g \leq 0 \\ -\dfrac{1}{2\varepsilon} \lambda(x)^2 \, d\Gamma, & \lambda + \varepsilon g > 0. \end{cases} \tag{1}$$

where ε is the augmentation parameter. The displacement test functions for both domains are chosen from appropriate functional spaces and have to satisfy Dirichlet boundary conditions.

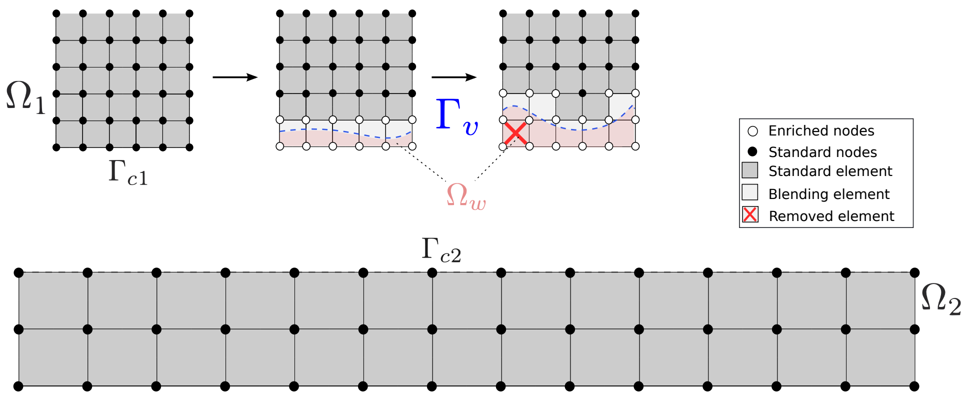

Figure 1. Material removal due to wear at the interface, modelled using a virtual interface Γ_v. Ω_w is the worn-out volume where the internal virtual work is not integrated.

The proposed method consolidates diverse and well established fields of the mortar domain decomposition methods and the X-FEM, working together to simplify and provide a general framework to treat efficiently interface phenomenon like contact and wear. In perspective the method will be taken forward into the realm of parallel computing, thus enabling to solve large and complex problems in a flexible and accurate way.

Examples

In Fig. 2 an example of mesh tying along a virtual surface Γ_v belonging to a coarse mesh Ω_0 in which a fine patch Ω_i containing a hole is inserted. In Fig. 3 the σ_{xx} stress component is

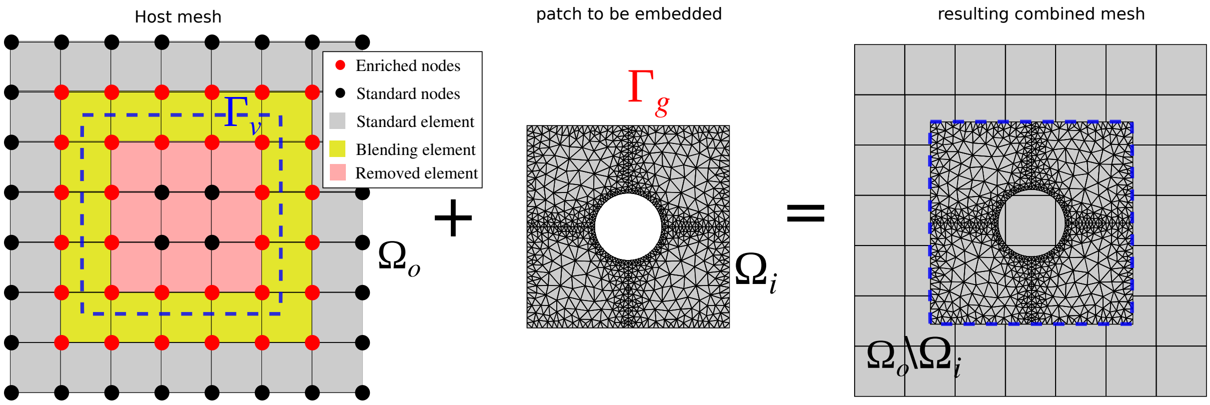

Figure 2. Tying between two mesh along internal virtual interface.

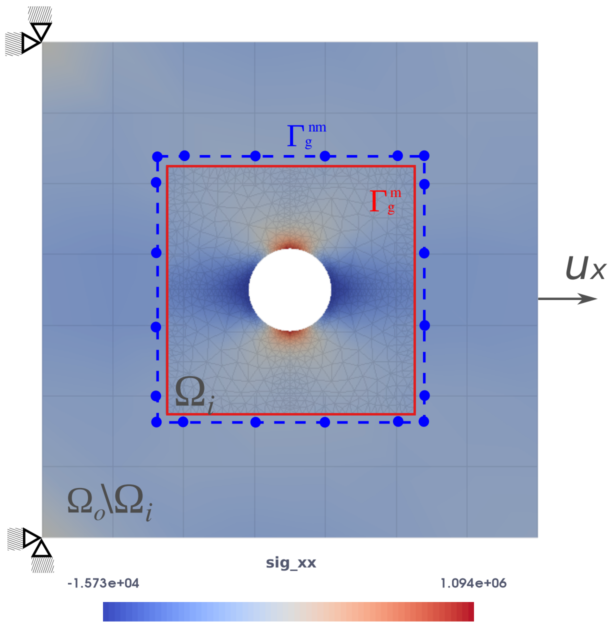

Figure 3. Stress field σ_{xx} in set-up presented in Fig. 2.

presented: a rather smooth transition between two stress fields across the interface is presented. In Fig. 4 another example

Conclusions

The presented coupling between the mortar method (or in general between any face-to-face method) and the X-FEM method enables to treat in an accurate and robust fashion complex problems arising in various contact applications related to sub-mesh surface geometries and wear evolution.

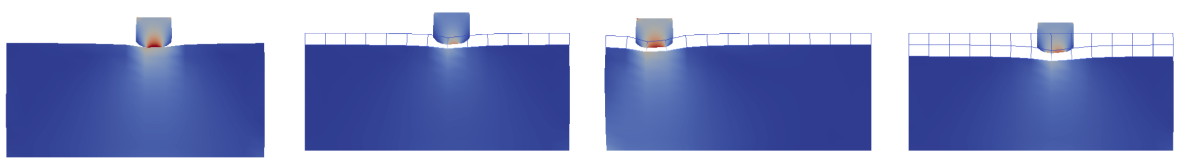

Figure 4. Example of contact along a virtual surface uniformly penetrating in the solid.

Acknowledgements

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