Two-Phase Model-Reduction for Two-Scale Simulations of Components

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Micro Abstract

The Nonuniform Transformation Field Analysis (NTFA) has been introduced by Suquet et al. to make two-scale simulations of components feasible for industrial sized problems. To reduce the computational complexity of the "offline" phase of the NTFA, the micro-solver applies the recently introduced composite voxel technique. The predicted effective response will be compared to both the original NTFA method using micro-scale simulations at full resolution and direct simulations.

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Introduction

For the simulation of industrial components made of composite materials, model order reduction (MOR) methods are commonly applied. These methods can be split into an "offline" and "online" phase. In the "offline" phase an effective material law for the macro-scale is derived by micro-scale simulations. In the "online" phase only ordinary differential equations for the reduced variables need to be solved to obtain the stress response of the composite material at each integration point of the component simulation.

Since the microstructure is different at each integration point the necessary precalculations in the "offline" phase depend linearly on the number elements in the mesh of the component. If plastic effects are to be taken into account for the component simulation, the computational effort for the "online" phase of the hybrid model order reduction method of Fritzen and Leuschner [1] also scales cubically with the resolution of the micro-scale.

In this article the composite voxel technique will be combined with the NTFA-TSO model of Michel and Suquet [5] a well as the hybrid model of Fritzen and Leuschner [1] to accelerate two-scale simulations of components with elasto-viscoplastic deformations.

1 Combining MOR methods with the composite voxel technique

To combine the composite voxel technique for nonlinear material behavior [4] with the above mentioned model order reduction methods, the precalculations are performed at quarter resolution by combining the FFT-based method of Moulinec and Suquet [6] with the discretization of Willot [7,8] (trilinear hexahedral elements with reduced integration) and laminate composite voxels or stress volume averaging.

As test example we reproduce the metal-matrix composite example in Section 7 of [5] where the constituents are modeled as generalized standard materials (GSM) [2,3]. To assess the potential of the composite voxel technique for accelerating the MOR methods, the microstructure was drastically simplified, see Figure 1.



Figure 1. Geometry for single endless fiber.

1.1 Results

In the example of Michel and Suquet the reproducibility of the Hybrid MOR method as well as their NTFA-TSO method is checked for an uniaxial tensile test. To be more precise, the loading curve for the reduced model is part of the snapshots performed in the precalculations. In Figure 2 and 3 the same validation is done for the combination of both MOR methods with laminate composite voxels and stress volume averaging at quarter resolution.

In the first line of Figure 2 the predictions of the original MOR methods are compared with direct simulations. In the left column the stress response in loading direction is shown whereas the right column shows the strain orthogonal to the loading direction. Both MOR methods reproduce the direct simulations almost perfectly. The second line shows the prediction of the MOR methods, when using precalculations at quarter resolution with stress volume averaging. In this case the effective stress response predicted by both MOR methods is not even qualitatively correct. By switching to laminate composite voxel the Hybrid MOR method gives the correct effective strain response but underestimates the stress response. Surprisingly, the NTFA-TSO predicts both the effective strain as well as the effective stress very good.

Figure 3 shows the predicted evolution of the plastic strains for both MOR methods. As for the effective strain and stress the prediction of the original MOR methods are almost perfect. In combination with volume stress averaging at quarter resolution the Hybrid MOR method overestimates the equivalent plastic strain by a factor of three and slightly overestimates the effective backstress. On the other hand the NTFA-TSO method predicts both the equivalent plastic strain as well as the effective backstress quite good. In combination with laminate composite voxel both MOR methods give better predictions. While the Hybrid MOR still overestimates the equivalent plastic strain by a factor of two, the NTFA-TSO shows the same solution quality as when using micro-scale simulations at full resolution.

Conclusions

Combining MOR methods with composite voxels at quarter resolution reduces the computational effort during the "offline" phase approximately by a factor of 4 * 4 * 4 = 64. Only for the Hybrid MOR method the same holds true for the "online" phase. Combining the NTFA-TSO with laminate composite voxel does not deteriorate the quality of the predicted effective response (for this reproducibility test). Surprisingly, this does not hold true for the Hybrid MOR method.



Figure 2. Comparison of MOR methods with direct simulations for endless fibers. First line: Full resolution. Second line: Stress volume averaging at quarter resolution. Third line: Quarter resolution with laminate composite voxel.

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Figure 3. Comparison of MOR methods with direct simulations for endless fibers. First line: Full resolution. Second line: Stress volume averaging at quarter resolution. Third line: Quarter resolution with laminate composite voxel

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