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Substituting FE analysis of cyclic processes by a space-time reduced order model

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Micro Abstract
The huge computational costs make classical Finite Element (FE) simulations of nonlinear structural problems subjected to long-term or e.g. cyclic loading a challenging task. One approach to tackle this is through Model Order Reduction (MOR). Space-time MOR leads to a low-dimensional nonlinear system of equations which is solved in coarse time intervals, e.g. for each load cycle. Thus, remarkable computational saving in terms of CPU time and memory space are attained.

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Introduction

Viscoelastic materials are widely used in industry, e.g. in aerospace, automotive and civil engineering. In many applications viscoelastic materials are exposed to long-term loading conditions. More precisely, rather slow creep processes and fast (generally cyclic but not necessarily harmonic) processes with many repetitions are found.

It is challenging to perform classical finite element (FE) simulations of time-dependent nonlinear problems in the presence of long-term loading: Overly many time increments may be needed and the amount of information that needs to be stored can be substantial. In order to reduce the computation time of long-term processes in the presence of (strongly) nonlinear viscoelastic effects and for rather general boundary conditions (i.e. not limited to cyclic loadings) , the authors propose the use of reduced order models, more precisely of a reduced basis (RB). While RB methods are often used to compress spatial information at given time instances (e.g. the displacement field and the internal variables at time t) [e.g., 2, 3], our current approach seeks ansatz functions that work directly in the space-time domain. Each basis function of the RB gives the full time-dependent field information on the entire spatial domain. The idea of using space-time solution techniques is certainly not new in nonlinear solid mechanics and reference to a long and rich history can be made. Noteworthy contributions have been developed under the umbrella term LATIN (LArge Time INcrement) developed by Ladevèze and co-workers [e.g. 4, to mention only few of the earlier references]. The relation between the proposed method and LATIN-PGD approach will be disscussed in the talk.

1 Modeling

A nonlinear version of the Zener or Maxwell composed of a linear base elasticity in parallel to several nonlinear Maxwell elements that exhibit a power-law creep behavior under isochoric loading is used while the volumetric response is assumed linear (see Fig.1). In the current work the constitutive modeling of the nonlinear viscoelastic solid is realized via internal state variables within the framework of Generalized Standard Materials[GSM, e.g. 1]. The viscoelastic materials are modeled via quadratic free-energy and nonquadratic dissipation potential. Although this might appear restrictive, the framework contains also viscoplasticity and extensions to

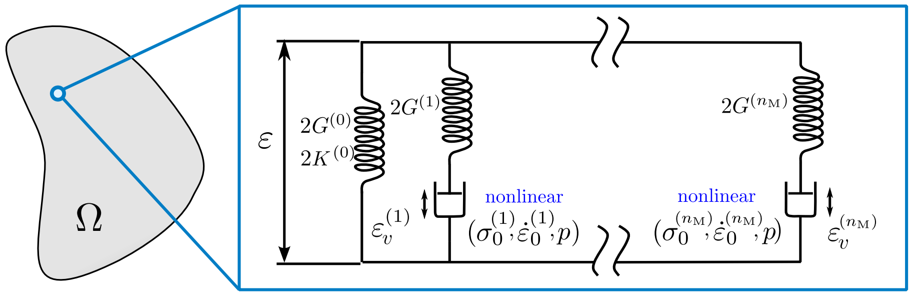

Figure 1. Schematic representation of the nonlinear viscoelastic model under consideration

nonquadratic potentials and to kinematically nonlinear settings without major modifications of the underlying methodology. Notably the considered formulation is almost identical to Perzyna-type viscoplasticity of plastically incompressible materials.[e.g. 5]

2 Space-time reduced order model

In the considered framework the total and viscous strain fields defined as functions of space-time are the unknowns that should be reduced. The snapshot Proper Orthogonal Decomposition (POD) can be used in order to obtain a reasonable reduced basis (RB) for parameter-dependent problems: first, "snapshots" are gained from the solution of high-fidelity (i.e. unreduced) problems at given parameters (here: different loadings). Second, the obtained data is processed: a correlation matrix is constructed and the leading eigenpairs are used to recombine the snapshots in order to construct the RB. Here the focus is on the methodology for the computation of the reduced coefficients in a space-time reduced setting and not on the construction of the RB.

2.1 Reduced basis ansatz for space-time problems

Consider that a long-term process with a time-dependent vector-valued variable $\underline{\xi}(t) \in \mathbb{R}^d$ is of interest. Our strategy is two-fold: first, the natural time (t) is partitioned into intervals or cycles of variable length (see Fig.2). For simplicity the time discretization is assumed equidistant

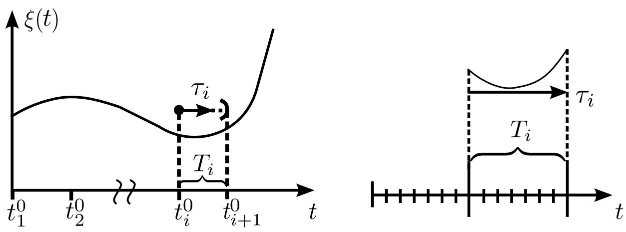

Figure 2. Natural time t and microscopic time τ_i in the ith time interval (left: natural time; right: related micro-time)

within each of the intervals. Then the natural time coordinate t within the $i-$th time interval is expressed via a new "micro-time" τ via

$$t = t_i^0 + \tau \qquad \tau \in [0, T_i). \tag{1}$$

We do not intend to predict the structural behavior over the entire simulation time (i.e. over thousands of load cycles) via a single reduced computation. Instead the problem is recast into interval- or cycle-wise defined micro-problems and a space-time reduced model for the solution on the individual intervals/cycles with appropriate transition conditions guaranteeing continuity of the state variables is proposed [6]. This is a difference to several proposals related to the proper generalized decomposition, where the full space-time solution is usually predicted in single computation. In our reduced model we explicitly permit for variable loading conditions.

We assume that the RB and the reduced approximation of $\underline{\xi}(t)$ in a single load cycle i ($t \in [t_i^0, t_{i+1}^0)$) are given by the linear relation

$$\underline{\xi}^{(i)}(t) = \underline{\underline{\mathcal{L}}}_{\xi}^{(i)}(\tau)\, \underline{y}^{(i)}, \qquad \tau = t - t_i^0, \qquad \underline{y}^{(i)} \in \mathbb{R}^m, \tag{2}$$

$$\underline{\underline{\mathcal{L}}}_{\xi}^{(i)} : \begin{cases} [0, T_i) & \to \mathbb{R}^{d\times \mathrm{m}}, \\ \tau & \to \underline{\underline{\mathcal{L}}}_{\xi}^{(i)}. \end{cases} \tag{3}$$

In the above definitions $\underline{\underline{\mathcal{L}}}_{\xi}^{(i)}$ is the RB in time and $\underline{y}^{(i)}$ is the vector of reduced coefficients. In order to avoid the dependency of the RB on the time interval counter i while preserving variable interval (or cycle) length T_i, the rescaled time coordinate $\widetilde{\tau}$ is introduced via

$$\widetilde{\tau} = \frac{\tau}{T_i}, \qquad \widetilde{\tau} \in [0, 1). \tag{4}$$

2.2 Continuity of the reduced fields

As schematically shown in Fig.2 the function $\underline{\xi}(t)$ is assumed to be time-continuous. This implies that the approximation via the RB approach should also be time-continuous, i.e.

$$\underline{\xi}^{(i)}(\tau \to T_i) \overset{!}{=} \underline{\xi}^{(i+1)}(\tau = 0). \tag{5}$$

Therefore two modifications of the RB approximation are suggested [6]:

- The modes are zero at the beginning of the time interval:

$$\underline{\underline{\mathcal{L}}}_{\xi}^{(i)}(\tau = 0) = \underline{\underline{\mathcal{L}}}_{\xi}^{0}(\widetilde{\tau} = 0) = \underline{0}. \tag{6}$$

- The reduced approximation is obtained from (i) the time-dependent RB approximation and (ii) a constant offset, i.e. via the affine relation

$$\underline{\xi}^{(i)}(\tau) = \underline{\underline{\mathcal{L}}}_{\xi}^{(i)}(\tau)\, \underline{y}^{(i)} + \underline{\xi}^{(i-1)}(T_{i-1}). \tag{7}$$

2.3 Reduced equilibrium conditions for the space-time reduced model

The reduced basis represents the behavior during a complete time interval/cycle through the reduced unknowns. Once the reduced ansatz is established, it remains to define mechanically motivated conditions for these reduced unknowns. These missing relations are found by enforcing the stationary conditions of the time-continuous system in the weak sense over the current time interval.

3 Validation methodology

In order to investigate the accuracy and performance of the proposed method different discrete problems subjected to various types of loading conditions are considered. Two important points are investigated:

- **reproduction capability**
  Can the reduced computations reproduce the snapshot data up to the expected degree of accuracy?

- **prediction capability**
  What is the accuracy for loading conditions different from the offline phase (i.e. different from the training data)? This concerns load frequencies (or loading/unloading times), the sequence of frequencies, the shape of the loading and the load amplitudes, i.e. the generalization of the load spectra from the offline phase.

Conclusions

A framework for nonlinear viscoelasticity is proposed . Other than within classical FE discretizations the internal variables are considered as additional unknowns of the (unreduced) FE problem. The related discretization remains nonstandard and has several implications:

- The local constitutive model reduces to a straight-forward function evaluation.
- In contrast to standard FE problems the nodal displacements and the internal variables are iterated simultaneously.
- No algorithmic tangent must be computed and most of the stiffness matrix is constant which allows for fast assembly procedures.

The space-time reduced order model is established through the following steps:

- a subdivision of the natural time into intervals (or cycles);
- time re-scaling of the RB in order to account for the true rate of the state defining variables;
- the careful consideration of the continuity of the reduced ansatz across time intervals;

The developed method is applied to discrete mechanical systems consisting of networks of springs and nonlinear dashpots. It is found that the reduced model using velocity-type test functions has better accuracy and, at the same time, need less iterations to converge. With respect to the numerical performance, the number of Newton Raphson iterations is low although all mechanical and constitutive variables during the entire time interval/cycle are iterated simultaneously by updating the reduced unknowns.

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