Proceedings of the 7th GACM Colloquium on Computational Mechanics
for Young Scientists from Academia and Industry
October 11-13, 2017 in Stuttgart, Germany

A simplified damage model for unilateral behavior of concrete

Ajmal Hasan Monnamitheen Abdul Gafoor[1]* and Dieter Dinkler[1]

Micro Abstract
For complex loadings of concrete structures as earthquakes, cyclic and dynamic conditions must be taken into account. Therefore a simplified damage model which uses an equivalent strain in terms of invariants of elastically predicted stresses is developed. The model is introduced by two history deformation parameters related to the equivalent strain in order to describe unilateral behavior of concrete. Thus the model can simulate the distinct behavior of concrete under monotonic/cyclic loadings.

[1]Institute for Structural Analysis, Technical University of Braunschweig, Braunschweig, Germany
***Corresponding author**: a.abdul-gafoor@tu-bs.de

Introduction

In the field of civil engineering concrete is the most commonly used construction material among others due to easy applications. Concrete generally exhibits ductile behavior under compression and brittle behavior under tension. Therefore, it is very essential to understand failure mechanisms of concrete under various loadings to ensure the safety of concrete structures against complete failure. In the past decades, several numerical models from elasto-plastic models, fracture models, elasticity based damage models to coupled plastic-damage models have been developed to simulate its deformation behavior. However the applications of these models are limited to particular loading conditions. The complex loading conditions as in case of earthquakes make the damage process even more complicated, if cyclic and dynamic loadings have not been considered.

In the context of continuum damage mechanics, a history deformation parameter is generally related to a local measure of deformation called as an equivalent strain describing the damage growth. The damage equivalent strain is proposed as Mazars strain [7], Modified von-Mises strain [10] and two equivalent strains for cracking and crushing respectively [6]. Even though the equivalent strain is modified in order to account for predicting the tensile and compressive behavior, these modifications are not yet fully capturing the initial elastic domain and ultimate stress domain. The numerical results are showing considerable differences, especially in bicompression and complex regions, while comparing with experimental data [12]. Moreover, the numerical predictions of the μ-model [6] are not smooth at all, although this model offers good results with few differences near bisector of bicompression region. Hence, the equivalent strain has to be defined appropriately to predict both tensile and compressive behavior of concrete, as it plays an essential role in the evolution of damage.

Nonetheless, strain-softening of the material eventually causes localization problems and thus leads to unacceptable results upon mesh refinement, if only conventional local continuum theories are used to model the material behavior. In order to overcome these difficulties, several regularization methods have been proposed to yield mesh-independent and well-posed solutions. Thus, nonlocal continuum models introducing an internal length scale by means of either integral forms [11] or implicit/explicit gradient forms [10] have become familiar. Owing to its convenience and straightforwardness in linearisation, the implicit-gradient formulation has been successfully implemented for intended results under various loading environments [1].

Therefore, the damage model which is previously developed under monotonic loading conditions [8] is enabled to account for unilateral behavior of concrete in this work. The proposed model uses a unified expression for the damage equivalent strain in three dimensional setting which is estimated from the predicted equivalent stress involved in Lubliner's failure criterion [4]. This equivalent strain is solely used to drive the evolution of effective damage through the history deformation parameter. The continuum model incorporates an implicit gradient method [10] to enrich the equivalent strain.

1 Nonlocal damage formulation

According to the principle of energy equivalence the constitutive relation for the elasticity based damage behavior of a material is written by equation (1), if $\boldsymbol{\sigma}$ and $\boldsymbol{\varepsilon}$ are the second-order tensors of Cauchy stress and strain respectively; $\mathbb{C}$ is the fourth-order elasticity tensor of the undamaged material. In order to describe the strain-softening behavior of concrete, a scalar/isotropic measure of effective damage D is introduced explicitly as a function of a history deformation parameter κ since D is generally governed by κ.

In a nonlocal damage model, κ is related to the distribution of damage strain (i.e. the nonlocal equivalent strain) $\bar{\varepsilon}_{eq}$ in the vicinity of the material point. Therefore, the nonlocal quantity $\bar{\varepsilon}_{eq}$ is approximated by the partial differential equation (7). In which, ∇^2 is the Laplacian operator and l_c is the characteristic internal length which is required to regularize the localization problem. The equation (7) is supplemented by an additional natural boundary condition (8). In case of a local model ($l_c = 0$), $\bar{\varepsilon}_{eq} = \varepsilon_{eq}$ and thus κ will be related to the local measure of deformation called as damage equivalent strain ε_{eq} estimated by the expression (3), where I_1 and J_2 are the first invariant of elastically predicted stress and second invariant of the deviatoric part of stress respectively; σ_{max} is the maximum principal stress and H is a Heaviside function, whereby $H = 1$ if $\sigma_{max} > 0$, else $H = 0$. The parameters α_L and β_L depending on the uniaxial tensile and compressive strengths of concrete as defined in [4].

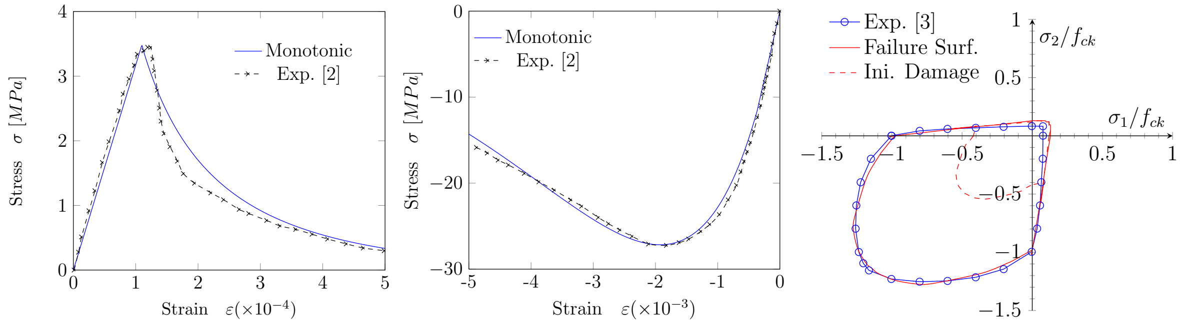
The possible damage growth can conveniently be decided based on the damage criterion f given by the equation (4). In order to account for the unilateral behavior of concrete, two independent history variables κ_t and κ_c are introduced here. Respectively their initial threshold values are set by κ_{0i} for tension ($i = t$) and compression ($i = c$). κ is described as a maximum deformation occurred during tension or compression loading path. The conditions of damage process can mathematically be expressed by the Kuhn-Tucker loading/unloading relations (9), where $(\dot{\ })$ represents the derivative of the variable with respect to time t. The consistency condition $\dot{f} = 0$ must always be valid during the damage process. The initial threshold values are used to limit the initial elastic region.

Table 1. Isotropic damage model

Description		Model Equations	
Constitutive law (energy equivalence)	:	$\boldsymbol{\sigma} = (1 - D(\kappa))^2 \mathbb{C} : \boldsymbol{\varepsilon}$,	(1)
Damage evolution law [9]	:	$D(\kappa) = 1 - \left[\frac{\kappa_0}{\kappa}\right]^{\beta_1} e^{-\beta_2\left[\frac{\kappa - \kappa_0}{\kappa_0}\right]}$,	(2)
Damage equivalent strain, ε_{eq}	:	$\varepsilon_{eq} = \frac{1}{E(1-\alpha_L)}\Big(\alpha_L I_1 + \sqrt{3J_2} + \beta_L H \sigma_{max}\Big)$,	(3)
Damage loading surface, f	:	$f = \bar{\varepsilon}_{eq} - \kappa \leq 0$,	(4)
History deformation parameter, κ	:	$\kappa = \kappa_t H + \kappa_c(1 - H)$,	(5)
in cracking ($i = t$) or crushing ($i = c$)	:	$\kappa_i = \mathrm{Sup}\big[\kappa_{0i}, \max\ \bar{\varepsilon}_{eq}\big]$	(6)
Implicit gradient method	:	$\bar{\varepsilon}_{eq} - l_c^2 \nabla^2 \bar{\varepsilon}_{eq} = \varepsilon_{eq}$,	(7)
Natural boundary condition	:	$\nabla \bar{\varepsilon}_{eq} \cdot \boldsymbol{n} = 0$,	(8)
Kuhn-Tucker relations	:	$f \leq 0, \quad \dot{\kappa} \geq 0, \quad f\dot{\kappa} = 0.$	(9)

2 Model validation

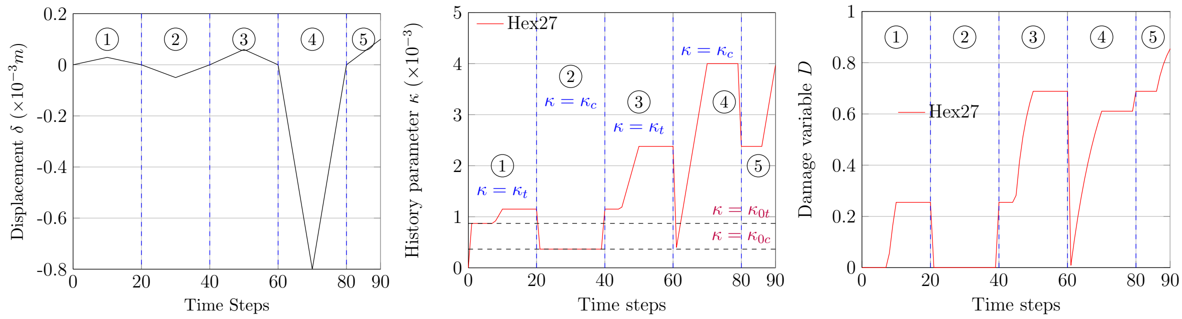
The proposed simplified damage model has been implemented in a finite element program (codeBlue) to solve several problems. A single-brick element mesh of size $200 \times 200 \times 200\ mm$ with 27 Gauss integration points is used in testing the model. Numerical results agree well when compared with experimental data under monotonic uniaxial and biaxial loadings (see Figure 1).



(a) Uniaxial tension **(b)** Uniaxial compression **(c)** Biaxial tests

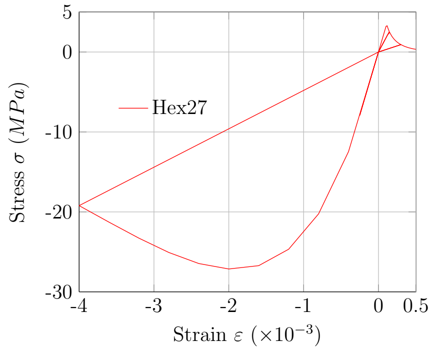
Figure 1. Numerical simulation of model under monotonic loadings

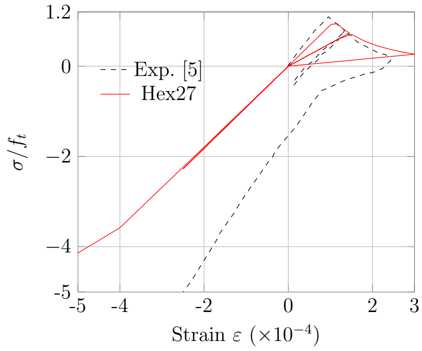
In order to investigate the effect of damage under uniaxial cyclic tension-compression loading, the displacement history as displayed in Figure 2a is used to be cyclic with an increasing magnitude as an imposed loading. Convergence is very quickly obtained. The material and model parameters used in these tests are provided in Table 2. Hex27 element is enhanced



(a) Imposed loading history **(b)** Evolution of history paramater **(c)** Evolution of damage

Figure 2. Numerical simulation of model under cyclic tension-compression





E	$= 31700\,\text{MPa}$
f_{ck}	$= 27.6\,\text{MPa}$
f_t	$= 3.48\,\text{MPa}$
ν	$= 0.21$
β_1	$= 0.85_t,\ 0.0_c$
β_2	$= 0.18_t,\ 0.095_c$
κ_{0t}	$= f_{ck}/E$
κ_{0c}	$= 10 f_t/3E$
l_c	$= 90\,\text{mm}$

Figure 3. Unilateral behavior under given loading (Figure 2a)

Figure 4. Normalized Unilateral behavior compared to [5]

Table 2. Material & Model Parameters

by nonlocal enrichment. Nonlocal variable $\bar{\varepsilon}_{eq}$ is adopted as an additional unknown at each corner nodes of the element along with displacements as primary unknowns. The appropriate selection of characteristic length l_c greater than the distance of two Gauss points leads to simulate the tension-compression behavior of concrete completely. As seen in Figure 2b κ_t or κ_c is monotonically increasing during tension-compression loading/unloading processes respectively. Figure 2c depicts a monotonic increase of the effective damage D. It is also observed that D

becomes zero after the initial tensile loading (cycle 1) and then entering into the first phase of compression (cycle 2) with initial stiffness. But D is recovered once the material experiences tension in reloading (cycle 3). Thus, the model describes the unilateral behavior quite well as shown in Figure 3. Furthermore, the normalized unilateral responses of numerical model as well as the experiments [5] are illustrated in Figure 4. Numerical response agrees well with the experimental one showing a full recovery of initial stiffness. But there are certain discrepancies due to the fact that the permanent inelastic strains are not taken into account in the model.

Conclusions

The proposed 3D model is simplified using a unified equivalent strain as a driving force to the evolution of effective damage variable to describe the crack-opening/closure effects on microcracks under cyclic loadings. The implicit gradient method for nonlocal enrichment is incorporated. The model captures the unilateral behavior of concrete and also recovers a full initial stiffness for compression region under the absence of inelastic strains. Thus, the numerical predictions of the model exhibit good agreement with the experimental results. Nevertheless, further work includes the inelastic evolution and damage induced anisotropy.

Acknowledgements

This work is supported by DAAD in the framework of Research Grants (No.:57048249).

References

[1] F. Cramer, U. Kowalsky, and D. Dinkler. Coupled chemical and mechanical processes in concrete structures with respect to aging. *Coupled systems mechanics*, 3:53–71, March 2014.

[2] I. D. Karsan and J. O. Jirsa. Behavior of concrete under compressive loadings. *Journal of the Structural Division*, 95(12):2543–2564, 1969.

[3] H. Kupfer, H. K. Hilsdorf, and H. Rusch. Behavior of concrete under biaxial stresses. In *Journal Proceedings*, volume 66, pages 656–666, 1969.

[4] J. Lubliner, J. Oliver, S. Oller, and E. Onate. A plastic-damage model for concrete. *International Journal of solids and structures*, 25(3):299–326, 1989.

[5] J. Mazars, Y. Berthaud, and S. Ramtani. The unilateral behaviour of damaged concrete. *Engineering Fracture Mechanics*, 35(4):629–635, 1990.

[6] J. Mazars, F. Hamon, and S. Grange. A new 3d damage model for concrete under monotonic, cyclic and dynamic loadings. *Materials and Structures*, 48(11):3779–3793, 2015.

[7] J. Mazars and G. Pijaudier-Cabot. Continuum damage theory-application to concrete. *Journal of Engineering Mechanics*, 115(2):345–365, 1989.

[8] A. H. Monnamitheen Abdul Gafoor. Isotropic damage model using a new damage equivalent strain. *Report of 4th IABSE Young Engineers Colloquium (YEC 2017) Bochum*, pages 38–39, 2017.

[9] A. H. Monnamitheen Abdul Gafoor and D. Dinkler. Influence of deformation history parameters on softening behaviour of concrete. *PAMM*, 16(1):155–156, 2016.

[10] R. Peerlings, R. De Borst, W. Brekelmans, and M. Geers. Gradient-enhanced damage modelling of concrete fracture. *Mechanics of Cohesive-Frictional Materials*, 3(4):323–342, 1998.

[11] G. Pijaudier-Cabot and Z. P. Bazant. Nonlocal damage theory. *Journal of Engineering Mechanics*, 113(10):1512–1533, 1987.

[12] F. Ragueneau, R. Desmorat, and F. Gatuingt. Anisotropic damage modelling of biaxial behaviour and rupture of concrete structures. *Computers & Concrete*, 5(4):417–434, 2008.