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Viscoplastic reduced order homogenization using mixed formulations

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Micro Abstract
The effective response of viscoplastic composites is studied by means of two reduced order models that use a mixed variational formulation developed in the framework of the Transformation Field Analysis : the pRBMOR and the MxTFA. The MxTFA is based on a mixed variational formulation of viscoplasticity involving stress and plastic multiplier as independent variables, while the pRBMOR assumes non uniform inelastic strain and hardening modes. Examples show the capabilities of the two approaches.

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Introduction

The use of composite materials has significantly increased in the last decades thanks to their mechanical properties. When it comes to the design of a composite structure, a multiscale problem needs to be solved, where at the macroscale (i.e. the design scale) the material can be considered as homogeneous, while at the microscale it is characterised by a heterogeneous microstructure. The latter has a significant influence on the mechanical behaviour of the material at the macroscale, especially in case of nonlinear behaviour of the constituents. The non linear phenomena that characterize the constituents (e.g. damage, viscoelasticity and fracture) need to be properly modelled in order to have an accurate description of the material behaviour. For this reason a reliable design tool is required. The so-called two-scale finite element method or FE^2 (e.g. [4]) is a possible solution, where both the microscale and macroscale problems are solved employing nonlinear finite elements. This leads to high computational costs due to the number of internal variables. Model order reduction techniques are an alternative to this method, since they allow for the solution of the problem by reducing the number of internal variables. Among them, the Transformation Field Analysis (TFA) [3] is an interesting approach. Nonlinearities within the heterogeneous microstructure are described by means of the inelastic strain. Other TFA-based schemes were proposed, differing mainly in the assumption on the inelastic strain distribution and in the evolution of the internal variables. Within this framework, the aim of this study is to compare two TFA-based reduced methods for the analysis of nonlinear composites: the pRBMOR cf. [6,7] and the MxTFA cf. [2]. Both techniques employ nonuniform transformation fields [8] and are based on a mixed variational formulation. The differences between these techniques will be analyzed in terms of mode selection, pre-analyses, evolution of reduced DOFs and effective stress computation.

1 Reduced order models: pRBMOR and MxTFA

In the following two mechanical homogenization procedures are presented that start from a parameterization of either internal state variables and/or stress fields.

1.1 Mode selection

As stated before, the techniques are all NTFA-based, i.e. they assume nonuniform inleastic strain distribution over the domain. Intuitively, the choice of the approximating modes will have a significant impact on the accuracy of the solution. A first difference in the choice of the modes is that the MxTFA assumes a domain partitioning into subsets, and defines the inelastic strain distribution subset-wise, while pRBMOR defines the inelastic modes on the whole RVE. The MxTFA defines self-equilibrated stress fields and a plastic multiplier approximation, and the inelastic strain approximation is consequently derived. A different approach is adopted by the pRBMOR that defines the inelastic modes by simulating the nonlinear response of the UC along prescribed loading conditions.

1.2 Pre-analyses

The pre-analyses are the offline-phase analyses required to obtain all the operators that are necessary for the nonlinear homogenization analysis. The mode selection has a direct impact on these analyses, since the number of modes is one of the parameters affecting the number of pre-analyses. Thanks to the choice of analytical strain modes, the MxTFA requires linear elastic pre-analyses aimed at computing the strain localization operators only. The number of analyses depends also on the number of subsets. The pRBMOR requires nonlinear pre-analyses in order to pre-compute the inelastic modes and to obtain the strain localization operators: the number of analyses in this case is affected also by the choice of the numerical training.

1.3 Evolution of reduced DOFs

Another difference relies in the resolution of the evolution problem. In fact, although the investigated homogenization strategies have large parts in common (for instance the self-equilibrated stress fields inspired by the TFA/NTFA), the biggest difference is found in terms of the evolution of the reduced degrees of freedom. The evolution of the reduced vectors of the pRBMOR is motivated from a mixed variational formulation that leads to a saddle-point problem (see [5, 6] for details). As for the MxTFA, the independent variables of the problem (i.e. the stress and plastic multiplier parameters) are obtained by enforcing the weak fulfillment of the compatibility equation and of the evolution of the plastic multiplier for every subset. More details regarding the numerical procedure can be found in [2].

1.4 Effective stress

The effective stress describes the material response at the macroscale. This quantity can be computed only once the evolution problem is solved at the microscopic scale in terms of internal variables. The latter are different among the techniques: for the pRBMOR they are the inelastic strain and hardening coefficients and for the MxTFA the stress and plastic multiplier parameters. For this reason, the effective stress is computed differently. According to pRBMOR, this quantity is recovered from the strain and inelastic strain fields. Likewise in mixed-stress finite elements approaches, the MxTFA computes the effective stress directly from the stress parameters.

2 Numerical results

The UC consists in a spherical ceramic inclusion embedded in a viscoplastic matrix, with volume fraction equal to 17.3%. The material properties as well as the details regarding the homogenization analyses are reported in [1]. The UC is subjected to a loading/unloading involving both normal and shear strains as detailed in Table 1. Figures 1 and 2 shows the comparison of the results between the FE analysis and the reduced model analyses. Both models are able to reproduce the overall response of the UC for both the normal and the shear stress components.

Table 1. Load case.

Time [s]	$\bar{\varepsilon}_{xx}$	$\bar{\varepsilon}_{yy}$	$\bar{\varepsilon}_{zz}$	$2\bar{\varepsilon}_{xy}$	$2\bar{\varepsilon}_{yz}$	$2\bar{\varepsilon}_{xz}$
0	0	0	0	0	0	0
1	0	0	0	$\sqrt{2}\cdot 0.01$	0	0
2	0.01	0.01	0.01	$\sqrt{2}\cdot 0.01$	0	0
3	0.01	0.01	0.01	0	0	0
4	0	0	0	0	0	0

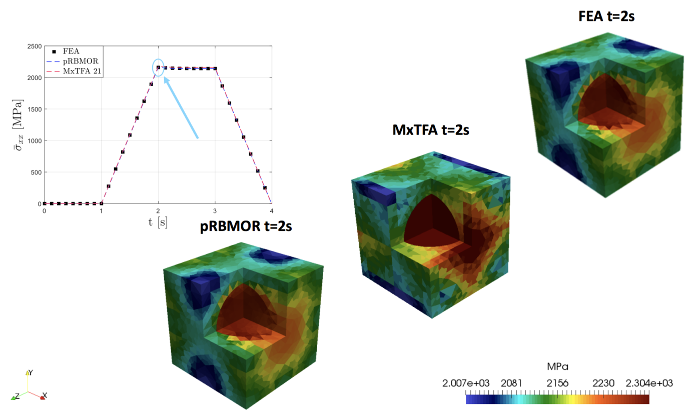

Figure 1. Numerical results: normal stress.

Conclusions

Two different homogenization techniques for the analysis of viscoplastic composite media were compared: the pRBMOR, the and the MxTFA. These techniques developed in the framework of the NTFA [8]. Moreover, both employ a mixed variational formulation for the resolution of the micromechanical problem. The key features as well as the differences of the techniques are described. The numerical results show the ability of the two reduced models to predict the effective behavior of 3D periodic composites characterized by nonlinear behavior under complex loading/unloading histories.

References

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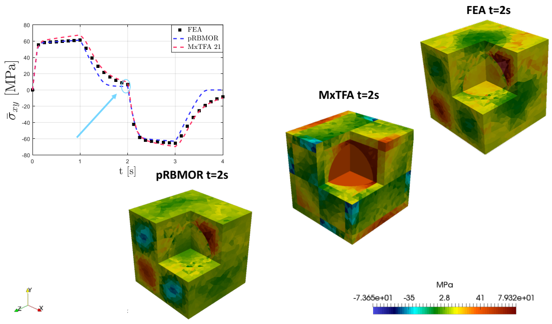

Figure 2. Numerical results: shear stress.

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