**Proceedings of the 7th GACM Colloquium on Computational Mechanics
for Young Scientists from Academia and Industry
October 11-13, 2017 in Stuttgart, Germany**

Application of the Phase-field Method for Crack Approximation on a Split-Hopkinson-Pressure-Bar Experiment

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Micro Abstract
The numerical simulation of fracture in brittle material remains a challenging task, both with respect to the suitable approximation technique as well as to the appropriate calibration experiment. The phase-field method – a promising approach under ongoing development – is applied to the simulation of the complex behavior of a concrete specimen in a split-Hopkinson-pressure bar experiment focusing on the capabilities of the method to obtain realistic crack initiation, evolution and arrest.

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Introduction

The standard design of concrete structures bases on the consideration of static and quasi-static loading. The behaviour of concrete under such loadings is well known and often simplified to be linear elastic with a tensile strength of zero. While this remains reasonable considering standard loading cases, the evaluation of the safety of structures subjected to extraordinary loads like earthquake or impact is a more challenging task, that requires a comprehensive understanding of the load and its interaction with the structrue as well as the consideration of the nonlinear behaviour of concrete beyond the linear elastic regime.

One of the most severe outcomes of an excess load is the evolution of cracks. A crack may lead to fragmentation and can change the mode a structure is bearing loads. The prediction if and how such a structure may collapse as well as the evaluation of residual load bearing capabilities is a complex topic involving mechanics, material science and additional challenges in connection with the numerical approximation of the relevant processes. Furthermore, the evolution of cracks is influenced significantly by the way and the speed a load is applied. Therefore, the development of a numerical model, that is capable of considering all relevant aspects of crack evolution, has to be based on sophisticated study and evaluation of experiments. One of the most common experiments to study the rate dependent behaviour of a material is the split-Hopkinson-bar (SHB) experiment. In this contribution, the application of the compression test configuration of the SHB on a concrete material is presented and studied numerically. The main focus is on the approximation of the complex crack evolution by a phase-field crack approximation approach as well as the numerical difficulties that arise in the setup of the experiment itself.

Numerical approximation of transient fracture by a phase-field method

The numerical simulation of transient problems usually are based on the strong form of the balance of linear momentum

$$\rho\,\ddot{\mathbf{u}} - \operatorname{div}\boldsymbol{\sigma} = 0 \tag{1}$$

with density ρ, the vector of acceleration $\ddot{\mathbf{u}}$ and the stress tensor $\boldsymbol{\sigma}$, that originates out of a HAMILTONIAN principle

$$\delta \int_{t_1}^{t_2} \int_{\Omega} \mathcal{L} \, dV + \int_{\partial\Omega} \mathbf{t}^* \cdot \mathbf{u} \, dA \, dt = 0, \tag{2}$$

that describes the variation δ of a balance between an inner part, considering the integration of the LAGRANGIAN density $\mathcal{L}$ over the domain Ω, and an outer part, considering the integration of the dot product between a prescribed vector of stresses $\mathbf{t}^*$ and a vector of displacement $\mathbf{u}$ over the domain's boundary $\partial\Omega$. In the phase-field method, the LAGRANGIAN contains, beside the standard contributions of kinetic and strain energy density, ψ_{kin} and ψ_{eps}, respectively, an additional part ψ_{dis}, that contributes for the amount of energy, that is necessary to evolve a crack by

$$\mathcal{L} = \psi_{kin} - \psi_{eps} - \psi_{dis}. \tag{3}$$

Based on GRIFFITH's theory, published in [2], the amount of energy necessary for the formation of a crack may be calculated out of the multiplication of the fracture toughness $\mathcal{G}_c$ with the area of the crack Γ and is energetically in balance with a certain part of the strain energy ψ^+_{eps}, which is dissipated in the process of crack surface formation. At this point, an additional degree of freedom, the phase-field order parameter p, that accounts for cracked material at $p = 1$ and unbroken material at $p = 0$, and a scalar length scale parameter l are introduced in order to obtain a regularized, numerical measurement for the crack surface Γ_l by

$$\Gamma \approx \Gamma_l = \int_{\Omega} \frac{\psi_{dis}}{\mathcal{G}_c} \, dV = \int_{\Omega} \frac{1}{2l} \left(p^2 + l^2 \, |\nabla p|^2 \right) \, dV \tag{4}$$

and, furthermore, to achieve a numerical coupling between the dissipated energy and the according part of the strain energy density by a degradation function

$$\psi_{eps} = (1-p)^2 \cdot \psi^+_{eps} + \psi^-_{eps}. \tag{5}$$

This leads to the second strong form equation

$$2 \, (1-p) \, \psi^+_{eps} - \frac{\mathcal{G}_c}{l} \, p + \mathcal{G}_c \, l \, \nabla^2 p = 0, \tag{6}$$

that governs the evolution of cracks based on an energetic consideration.

There are two common approaches to define the part ψ^+_{eps}, that contributes to the evolution of crack surfaces. The volumetric-deviatoric split, where

$$\psi^+_{eps} = \frac{3\lambda + 2\mu}{6} \cdot \langle \boldsymbol{\varepsilon} \colon \mathbf{1} \rangle^2_+ + \mu \cdot (\boldsymbol{\varepsilon}_D \colon \boldsymbol{\varepsilon}_D) \, , \tag{7}$$

employs the LAMÉ constants λ and μ, the second order identity tensor $\mathbf{1}$ and the bracket operator $\langle \bullet \rangle_{\pm} = (\bullet \pm | \bullet |) / 2$, is applied to a phase-field model e.g. by AMOR et al. in [1], and originates from standard damage mechanics assumptions, i.e. the strain tensor $\boldsymbol{\varepsilon}$ is split into volumetric and deviatoric components, assuming that a volumetric expansion as well as any deviatoric strain component may induce crack evolution. The second approach, that is introduced by MIEHE et al. in [5], proposes a spectral decomposition of the strain tensor by

$$\boldsymbol{\varepsilon} = \underbrace{\sum_i \langle \varepsilon_i \rangle_+ \, \mathbf{n}_i \otimes \mathbf{n}_i}_{\boldsymbol{\varepsilon}_+} + \underbrace{\sum_i \langle \varepsilon_i \rangle_- \, \mathbf{n}_i \otimes \mathbf{n}_i}_{\boldsymbol{\varepsilon}_-}, \tag{8}$$

where the principal strains are specified in terms of eigenvalues ε_i and according eigenvectors $\mathbf{n}_i$ and, in addition, categorised into tensile and compressive parts, $\boldsymbol{\varepsilon}_+$ and $\boldsymbol{\varepsilon}_-$, respectively. Furthermore, it is postulated, that the crack evolution is driven by

$$\psi^+_{eps} = \frac{\lambda}{2} \, \langle \boldsymbol{\varepsilon} \colon \mathbf{1} \rangle_+{}^2 + \mu \, \boldsymbol{\varepsilon}_+{}^2 \colon \mathbf{1}. \tag{9}$$

Split-Hopkinson-pressure-bar experiment

Experimental setup

The split-HOPKINSON-pressure-bar (SHPB) is an experimental setup to investigate the behaviour of a material under fast compressive loading. The experimental setup, consisting of a striker bar, an incident bar, the specimen and a transmitter bar, is shown schematically in Figure 1.

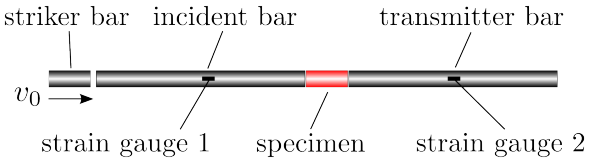
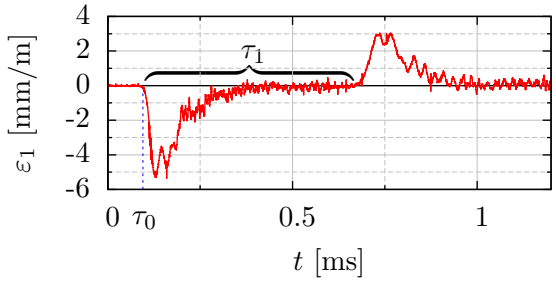


Figure 1. Schematic setup of the split-HOPKINSON-pressure-bar experiment

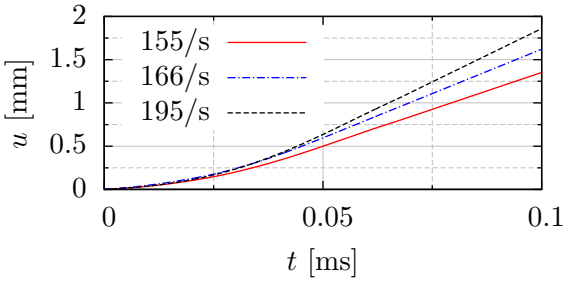
The striker, accelerated to the speed v_0, hits the incident bar, inducing a compressive wave, that is recorded as $\varepsilon_1(t)$ in strain gauge 1. At the interface between incident bar and specimen, a portion of the wave is reflected and can be recorded again at strain gauge 1 after time τ_1. A typical recording is shown in Figure 2a. The difference between incoming and reflected wave is assumed to be transmitted into the specimen as a compressive loading. The longitudinal displacement of the interface $u(t)$ is calculated by

$$u(t) = c \cdot \int_{\tau_0}^{t} \varepsilon_1(\tau) + \varepsilon_1(\tau + \tau_1)\, d\tau, \tag{10}$$

with $c = \sqrt{E/\rho}$ being the wave speed in the incident bar, and shown in Figure 2b for three different rates of loading.



(a) Typical record of longitudinal strain ε_1 in strain gauge 1 with respect to time t



(b) Longitudinal displacement u of the interface according to Eq. (10)

Figure 2. Evaluation of the strain gauge measurement into a displacement boundary for the contact surface between incident bar and concrete specimen.

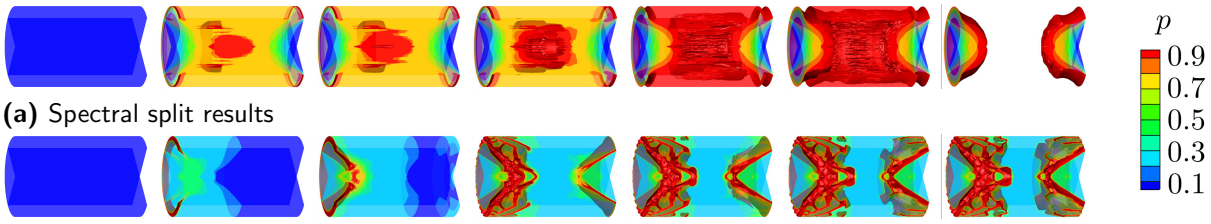
The compressive loading on the concrete leads to the evolution of cracks, while the residual fragments are a function of the loading speed. Figure 3 shows the initial specimen geometry and typical remainings of the structure after a compressive loading.

Figure 3. Geometry and typical fragments of a compression test with respect of the loading rate.

The experiments evaluated in this contribution have been performed in the OTTO-MOHR laboratorium of the Institute of Concrete Structures at TU Dresden. For a more detailed description and evaluation of the experiment, the reader is referred to [4].

Numerical simulation

The transient numerical simulation is restricted to the concrete specimen and focussed on the simulation and analysis of the fracture process, in order to evaluate the suitability of the crack approximation approach by the phase-field model. The experimental setup, i.e. the boundaries and loading, is simplified as a time dependent displacement boundary condition in longitudinal direction. The restriction of the lateral displacement plays a crucial role in the approximation of a realistic fracture pattern. The specimen is assumed to be of homogeneous linear elastic material with elastic modulus $E = 32$ GPa, POISSON ratio $\nu = 0.2$, and density $\rho = 2300$ kg/m^3. A fundamental assumption of the phase-field model is the argument, that the regularized crack surface Γ_l converges to the sharp crack Γ for $l \to 0$. Furthermore, it is shown in [3], that the characteristic length h of finite elements with linear shape functions should be connected to the length scale parameter by $h \approx l/2$. Therefore, a balance between an exact crack representation and a feasible computational time for the simulation has to be found. To this end, High-Performance-Computing has been applied for the simulation. Figure 4 shows typical results for the phase-field evolution for $l = 0.54$ mm.



(a) Spectral split results

(b) Volumetric deviatoric split results

Figure 4. Crack evolution represented with isosurface at $p = 0.95$ and blanked elements for $p > 0.95$.

Conclusions

The phase-field crack approximation is a powerful approach to investigate the process of fracture in brittle material under fast loading scenarios. Special care has to be taken for the correct numerical approximation of the boundary and loading, as they have a major influence on the results in dynamic crack propagation examples.

Acknowledgements

The authors gratefully acknowledge the financial support of "Deutsche Forschungsgemeinschaft" under grant KA 1163/19. Furthermore, we acknowledge the provision of the experimental data by the Institute of Concrete Structures of TU Dresden and the access to the Bull HPC-Cluster by the ZIH of TU Dresden.

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