Computational modeling of fiber flow during casting of fresh concrete

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Micro Abstract

The Folgar-Tucker fiber orientation model coupled with weakly compressible Smoothed Particle Hydrodynamics is used to predict the spatial-temporal evolution of the probability density function of fiber orientation during process of casting of fiber reinforced concrete. The flow-able concrete-fiber mix is modeled as a viscous Bingham-type fluid. The model predictions qualitatively agree with fiber orientations observed in an L-box test with fibers suspended in transparent gel.

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Introduction

Fiber reinforced concrete (FRC) is a construction material, which allows to better control the material ductility and crack spacing as compared to standard reinforced concrete. However, despite its wide and established use, many of the uncertainties and crude assumptions about the material properties and behavior are still in effect. In regard to the important questions of orientation and distribution of the fibers in the concrete or mortar matrix, in most practical applications these are assumed to be isotropic and homogeneous respectively. If scale bridging material models are employed for the simulation of FRC structures (see, e.g. [7]), such assumptions are required to predict the properties of material in hardened state such as tensile and compressive strength, elastic properties, and ductility. Evidently, these assumptions come from inherent difficulty to measure or predict the real orientation and distribution of fibers.

The aim of this work is to provide a robust numerical tool to estimate the orientation state during casting of fresh concrete. The structure of the manuscript is as follows: in the first section, the rheology of fiber suspensions is presented, including regularized the Bingham-Papanastasiou fluid model [6] and fiber concentration dependent viscosity of concrete by Ghanbari & Karihaloo [5]. The next section deals with the weakly compressible SPH (WCSPH) method, used to model the free-surface flow of non-Newtonian fluid during the casting process. Subsequently, the fiber orientation model of Folgar & Tucker used in this work is explained [4]. This is followed by the results section, where a comparison between experimental and numerical simulation of the L-Box test is discussed.

Modeling rheological behavior of fiber suspension using SPH

When fibers are mixed with concrete, a change of rheological properties occurs. These properties depend on the concentration and, to some extent, on the orientation of the fibers in the mixture and may also vary spatially. Ghanbari & Karihaloo [5] proposed a model describing the dependence of the plastic viscosity on the fiber concentration. The model assumes a homogeneous distribution and an isotropic random orientation of fibers in the whole domain. This model has been validated on a large set of experimental samples, and for that reason it is adopted in this work. The numerical example investigated in this work employs a Carbomer-type polymer whose behavior can be well described by the Bingham fluid model. Papanastasiou [6] gave a continuous function of apparent viscosity μ_{app} as a function of shear rate and numerical regularization



Figure 1. (a) Shear stress in Bingham fluid. Influence of regularization parameter *m*. (b) Example of a 2-D particle distribution surrounding particle a. The radius of influence of the kernel (smoothing length) is denoted by h. (c) Definition of fiber orientation angles.

parameter. The equations for the apparent plastic viscosity due to addition of fibers $(\mu_{pl,app})$, the apparent viscosity due to yield stress effects μ_{app} and the resulting shear stress tensor are given as:

$$\mu_{pl,app} = \bar{\mu}_{pl}[(1-c) + \frac{\pi c r_e^2}{3ln(2r_e)}], \quad \mu_{app} = \mu_{pl,app} + \frac{\tau_y}{\sqrt{|\dot{\gamma}|}}(1 - e^{-m\sqrt{|\dot{\gamma}|}}), \quad \tau = \mu_{app}\dot{\epsilon}, \quad (1)$$

where $\bar{\mu}_{pl}$ is intrinsic plastic viscosity of the fluid, c is the volume fraction of the fibers and r_e is an aspect ratio of the fiber, $\dot{\gamma} = \dot{\boldsymbol{\epsilon}}_s : \dot{\boldsymbol{\epsilon}}_s$ is the second invariant of shear rate tensor $(\dot{\boldsymbol{\epsilon}}_s = \dot{\boldsymbol{\epsilon}} - \frac{1}{3} \text{tr} \dot{\boldsymbol{\epsilon}}, \ \dot{\boldsymbol{\epsilon}} = \frac{1}{2} (\nabla^T \mathbf{v} + \nabla \mathbf{v}))$ and m is the regularization parameter. The performance of the Eq. 1 and its dependence on regularization parameter m is demonstrated in Fig. 1a.

Smoothed Particle Hydrodynamics (SPH) is a mesh-free method formulated in the Lagrangian frame of reference, meaning that interactions and derivatives are all evaluated in a coordinate system attached to a moving fluid particle. In this work, an open-source implementation of weakly compressible SPH code DualSPHysics [3] has been used and extended for the purpose of simulating the fiber flow. These extensions include the modification of the solver in order to consider the Bingham type flow and the implementation of the Folgar-Tucker model to simulate the evolution of fiber orientation state. The main assumption in SPH is that any quantity $f(\vec{r})$ being a function of spatial coordinates can be approximated by an integral interpolation:

$$f(\mathbf{r}_a) = \int_{\Omega} f(\mathbf{r}) W(|\mathbf{r}_a - \mathbf{r}|, h) d\mathbf{r} \approx \sum_b V_b f_b W(|\mathbf{r}_a - \mathbf{r}_b|, h),$$
(2)

where Ω is the fluid domain, \mathbf{r}_a is the position of the particle where the function is evaluated, W is a weighting function or kernel (see Fig. 1b for illustration), h is the kernel support size (smoothing length) and $d\mathbf{r}$ is the differential volume element. In the discrete form, the summation is over all the particles b within the region of compact support of the kernel function, fixed by h. V_b is the volume corresponding to particle b, and f_b is equal to $f(\mathbf{r}_b)$. The governing equations for SPH simulations of weakly compressible fluid flow problems are Navier-Stokes equations written in a Lagrangian form, except that the pressure term is evaluated from an equation of state [3]. The material model is introduced through modeling of the term $\nabla \cdot \boldsymbol{\tau}$, using here the Bingham-Papanastasiou model (Eq. 1), and incorporating it into the SPH balance of momentum equation.

Fiber orientation model

Fibers move and orient in the flow according to forces and couples exerted on them by the fluid. In case of highly concentrated fiber suspensions, the orientations of the fibers are further often perturbed by contact and hydrodynamic interactions between fibers, and collisions with the aggregates and mold walls. In three dimensions orientation of a single cylindrical fiber is defined by two angles in spherical coordinates θ and ϕ or equivalently by a unit vector **p** in the direction of fiber axis with components: $p_1 = \sin\theta \cos\phi$, $p_2 = \sin\theta \sin\phi$, $p_3 = \cos\theta$, as illustrated in Fig. 1c. The model for fiber orientation used in the present work was proposed by Folgar & Tucker [4]. It describes the fiber orientation state by the probability density function (PDF). This PDF evolves in space and time according to the flow of the suspending fluid, taking into account interactions between fibers. The model as such is extremely computationally expensive. For this reason Advani & Tucker [1] proposed a re-formulation using orientation tensors. Orientation tensors are defined as even moments of the PDF of the fiber orientation in the following form:

$$\mathbf{a}_{n} = \oint_{\Omega} \underbrace{\mathbf{pp}...\mathbf{p}}_{n \text{ times}} \psi(\mathbf{p}) d\mathbf{p} = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \underbrace{\mathbf{pp}...\mathbf{p}}_{n \text{ times}} \psi(\mathbf{p}) d\theta d\phi.$$
(3)

To this end, the complicated evolution equation of orientation PDF is substituted by a set of simple evolution equations for orientation tensors. It has been shown, that the 2^{nd} order tensor contains sufficient directional information, such that the PDF can be approximated by it [1]. During the course of calculation of the evolution of the 2^{nd} order orientation tensor, a so-called closure problem arises, because a 4^{th} order orientation tensor is required for computation. In this work, the closure approach of Chung & Kwon is adopted [2]. The equation of evolution of 2^{nd} order tensor is given as:

$$\frac{d\mathbf{a}_2}{dt} = -(\dot{\boldsymbol{\omega}} \cdot \mathbf{a}_2 - \mathbf{a}_2 \cdot \dot{\boldsymbol{\omega}}) + \lambda(\dot{\boldsymbol{\epsilon}} \cdot \mathbf{a}_2 + \mathbf{a}_2 \cdot \dot{\boldsymbol{\epsilon}} - 2\mathbf{a}_4 : \dot{\boldsymbol{\epsilon}}) + 2C_I \dot{\gamma} (\mathbf{I} - 3\mathbf{a}_2), \tag{4}$$

where $\dot{\boldsymbol{\omega}}$ and $\dot{\boldsymbol{\epsilon}}$ are defined as functions of velocity gradient: $\dot{\boldsymbol{\omega}} = \frac{1}{2} (\nabla^T \mathbf{v} - \nabla \mathbf{v}), \, \dot{\boldsymbol{\epsilon}} = \frac{1}{2} (\nabla^T \mathbf{v} + \nabla \mathbf{v}), \, \lambda = \frac{(r_e^2 - 1)}{(r_e^2 + 1)}$ is the shape parameter of the considered body, r_e aspect ratio of the fiber, C_I is an empirical interaction coefficient. Finally, the orientation tensors can be easily visualized by ellipsoidal surfaces, where the eigenvalues denote radii, and the eigenvectors the axes orientations.

Results and Discussion

In this section a numerical simulation of the flow of a non-Newtonian polymer-water solution with fibers in L-box test is qualitatively compared to results from an experimental setup. Ultrasound gel mixed with water is used as a suspending material, due to its transparency and yield stress property. In Fig. 2f the moment-shear rate relations are given for polymer mixes of different proportions produced during preparation of experiment. The rheological measurements are conducted on Viscomat NT rotational viscometer. In this work a mixture of ultrasound gel and water in ratio 2:1 is used. The experimental setup is characterized by a box of dimensions $40 \times 30 \times 10$ cm, with a gate which opens vertically to the level of 10 cm above the bottom of the box (see Fig. 2a). After lifting the gate, the viscous fluid flows out rapidly up to the 32cm mark during approximately 0.6s (see Fig. 2b) and then abruptly slows down and continues creeping. This behavior is characteristic for viscous fluids with a pronounced yield stress. As the shear rate (and consequently shear stress) decreases below a certain threshold, the fluid develops an internal micro-structure which causes almost instantaneous "solidification" of the fluid. The volume fraction of fibers is estimated to be 0.5%, while the identified yield stress is $\tau_0 = 440 Pa$ and dynamic viscosity is $\mu = 1.5 Pas$. It can be noticed that the fibers in the region close to the walls tend to align parallel to the walls and in the zone between the walls they remain more or less randomly oriented with slight preference towards flow direction (see Fig. 2b, and Fig. 2e). A similar behavior is predicted by numerical model (see Fig. 2c, and Fig. 2d).

Conclusions

In this work a robust method to predict orientation states of fibers during casting process has been presented. The flowable concrete was represented by a regularized Bingham-Papanastasiou fluid model with a fiber concentration dependent viscosity. The Folgar-Tucker model used for the modeling of the fiber orientation during the flow process was combined with the weakly compressible SPH method. A comparison of the computational simulation results with the fiber



Figure 2. L-box test: (a) velocities of the simulated flow, (b) experiment, (c) fiber orientation state at time instant t = 0.6s, (d) top view of the orientation state after 0.6s, (e) experiment, (f) rheological characterization of Carbopol suspension and mixture of ultrasound gel with water using Viskomat NT.

orientation observed in experiments on a L-shape box test has demonstrated a qualitatively good agreement. However, the model should be enhanced by taking into account fiber dispersion, and the influence of spatially varying fiber concentration and orientation on the flow properties.

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