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Numerical investigation of hydraulic fracturing and borehole interaction under deep reservoir conditions using XFEM

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Micro Abstract
Hydraulic fracturing is a complex process, due to the interaction of the fracturing fluid with the surrounding deformable porous media, while the fracture is propagating. The fracture response is sensitive to in-situ stresses, fracture toughness or initial fracture angles. A hydro-mechanical XFEM model is introduced to simulate the fluid flow and deformation of the rock. Examples with multiple boreholes and initial fractures will investigate the influence of the aforementioned parameters.

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Introduction

Hydraulic fracturing is a technique for the stimulation of shale gas and deep geothermal reservoirs to increase the permeability of the underground. As monitoring of hydraulic fracturing in deep layers of the earth's crust is difficult, numerical simulations are required to investigate different scenarios and estimate the expected gain of subsurface energy.
In this work a fully coupled numerical algorithm for hydraulic fracturing in deformable saturated porous media is introduced and various examples with complex geometries and boundary conditions are analyzed. One method to simulate propagating cracks is the Extended Finite Element Method (XFEM), which has been extended to hydromechanically coupled problems, e.g. in [2] or [5] by introducing enhanced approximations for the displacement and the pressure field via the extended finite element method (XFEM) [1, 4]. The porous media flow is assumed to conform Darcy's law. In XFEM simulations the propagating crack is directly incorporated in the underlying mesh and no costly re-meshing is needed.
In the presented model this advantage is also used to incorporate the borehole geometry directly in the mesh with the XFEM. In contrast to the classical approach the fluid flow inside the crack is modelled explicitly and coupled with the pressure field of the porous media flow. [6] introduced this method and coupled the explicit fluid flow and the porous media flow with a pressure gradient approximation. In the approach of [6] no enrichment of the pressure field was taken into account. [3] proposed a space-time variant enrichment of the porous media pressure field in order to model the fluid flow in porous media more accurately. An alternative formulation of the XFEM, denoted as the interface-enriched generalized finite element method (IGFEM) introduced by [7], is used to approximate the pressure field of the porous media, using also linear shape functions for the pressure field across the crack. The advantage of the IGFEM is the straightforward definition of Dirichlet boundary conditions on non-matching meshes, like crack interfaces. The explicit fluid flow in the crack is coupled with the flow in the porous bulk medium by means of Lagrange multipliers by prescribing the pressure of the explicit fluid flow to the pressure of the porous media flow.

1 Governing Equations

The displacement field $\mathbf{u}$, the pressure field $\mathbf{p}_p$ in the bulk and the pressure field inside the fracture $\mathbf{p}_f$ are controlled by the balance of linear momentum and mass in association with Darcy's law and the Reynolds lubrication equation. After spatial discretization using finite element approximations, the weak form can be written in the form

$$\underbrace{\int_\Omega \mathbf{B}_u^T \mathbf{D} \mathbf{B}_u \, d\Omega}_{\mathbf{K}_u} \; \mathbf{u} - \underbrace{\int_\Omega \mathbf{B}_u^T \alpha \mathbf{m} \mathbf{N}_{p_p} d\Omega}_{\mathbf{Q}_{p_p}} \; \mathbf{p}_p - \underbrace{\int_{\Gamma_f} [\![\mathbf{N}_u]\!]^T \mathbf{n} \mathbf{N}_{p_f} d\Gamma}_{\mathbf{Q}_{p_f}^T} \; \mathbf{p}_f = \underbrace{\int_{\Gamma_t} \mathbf{N}_u^T \mathbf{t}^* \Gamma}_{\mathbf{F}_u} \tag{1}$$

$$- \underbrace{\int_\Omega \mathbf{B}_{p_p}^T \frac{\mathbf{k}_p}{\mu} \mathbf{B}_{p_p} d\Omega}_{\mathbf{K}_{p_p}} \; \mathbf{p}_p - \underbrace{\int_\Omega \mathbf{N}_{p_p}^T \frac{1}{Q} \mathbf{N}_{p_f} d\Omega}_{\mathbf{S}_{p_p}} \; \dot{\mathbf{p}}_p - \underbrace{\int_\Omega \mathbf{N}_{p_p}^T \alpha \mathbf{m}^T \mathbf{B}_u d\Omega}_{\mathbf{Q}_{p_p}^T} \; \dot{\mathbf{u}} \tag{2}$$

$$+ \underbrace{\int_{\Gamma_f} \mathbf{N}_{p_l}^T \mathbf{N}_{p_s} d\Gamma}_{\mathbf{L}_{p_p}^T} \; \boldsymbol{\lambda} = - \underbrace{\int_{\Gamma_q} \mathbf{N}_{p_p}^T \mathbf{n}^T \mathbf{q}^* d\Gamma}_{\mathbf{F}_{p_p}}$$

$$- \underbrace{\int_{\Gamma_f} \mathbf{B}_{p_f}^T \frac{w^3}{12\mu} \mathbf{B}_{p_f} d\Gamma}_{\mathbf{K}_{p_f}} \; \mathbf{p}_f - \underbrace{\int_{\Gamma_f} \mathbf{N}_{p_f}^T w \, c_f \mathbf{N}_{p_f} d\Gamma}_{\mathbf{S}_{p_f}} \; \dot{\mathbf{p}}_f - \underbrace{\int_{\Gamma_f} \mathbf{N}_{p_f}^T \mathbf{n}^T [\![\mathbf{N}_u]\!] d\Gamma}_{\mathbf{Q}_{p_f}} \; \dot{\mathbf{u}} \tag{3}$$

$$- \underbrace{\int_{\Gamma_f} \mathbf{N}_{p_l}^T \mathbf{N}_{p_f} d\Gamma}_{\mathbf{L}_{p_f}^T} \; \boldsymbol{\lambda} = - \underbrace{\mathbf{N}_{p_f} Q_0}_{\mathbf{F}_{p_f}}$$

$$\underbrace{\int_{\Gamma_f} \mathbf{N}_{p_s}^T \mathbf{N}_{p_l} d\Gamma}_{\mathbf{L}_{p_p}} \; \mathbf{p}_s - \underbrace{\int_{\Gamma_f} \mathbf{N}_{p_f}^T \mathbf{N}_{p_l} d\Gamma}_{\mathbf{L}_{p_f}} \; \mathbf{p}_f = \mathbf{0} \tag{4}$$

where $\mathbf{N}_u$, $\mathbf{N}_{p_p}$, $\mathbf{N}_{p_f}$, $\mathbf{N}_{p_l}$, $\mathbf{B}_u$, $\mathbf{B}_{p_p}$ and $\mathbf{B}_{p_f}$ are the sets of standard and enriched shape functions of the displacement field $\mathbf{u}$, the pressure fields $\mathbf{p}_p$ and $\mathbf{p}_f$ and the Lagrange multiplicator. The unknown Lagrangian coefficients $\boldsymbol{\lambda}$ are solved with the additional Equation 4 that carries information about the Dirichlet coupling between the two pressure fields in the bulk and the fracture, respectively. c_f is the fluid compressibility, $\mathbf{m} = [1\,1\,0]^T$ for 2D analyses. The boundaries Ω, Γ, Γ_f, Γ_t, and Γ_q are shown in Figure 1.

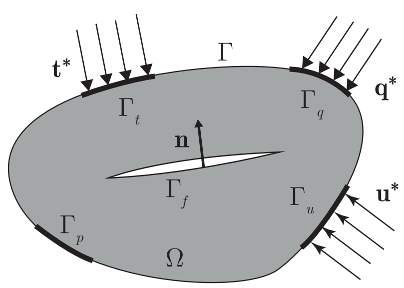


Figure 1. Geometry and boundary conditions of hydraulic fracturing problems in porous media

2 IGFEM Enrichment of the Pressure Field

A gradient jump in the porous media pressure field is described using the interface enriched generalized finite element method (IGFEM) as

$$p(x) = \underbrace{\sum_{i=1}^{n} N_i(x) p_{pi}}_{\text{standard part}} + \underbrace{\sum_{i=1}^{n_{en}} \psi_i(x) \alpha_i}_{\text{enriched part}} \tag{5}$$

where $\psi_i(x)$ are the enriched shape functions, α_i are the enriched degrees of freedom that lie on the intersection points of the fracture with the elements. As illustrated in Figure 2 the IGFEM is not based on the partition of unity, since the shape functions are identically zero on the element edges. Figure 2 shows how the enriched shape functions are generated by the standard shape functions of the children elements (1) and (2) in an intersected parent element. With regard to the nodal numbering of the children elements, for the situation shown in Figure 2, the enriched shape functions are given as

$$\psi_1(x) = N_1^{(2)}(x) + N_2^{(1)}(x), \qquad \psi_2(x) = N_1^{(1)}(x) + N_2^{(2)}(x).$$

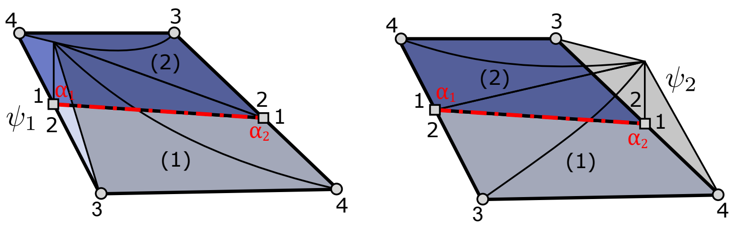
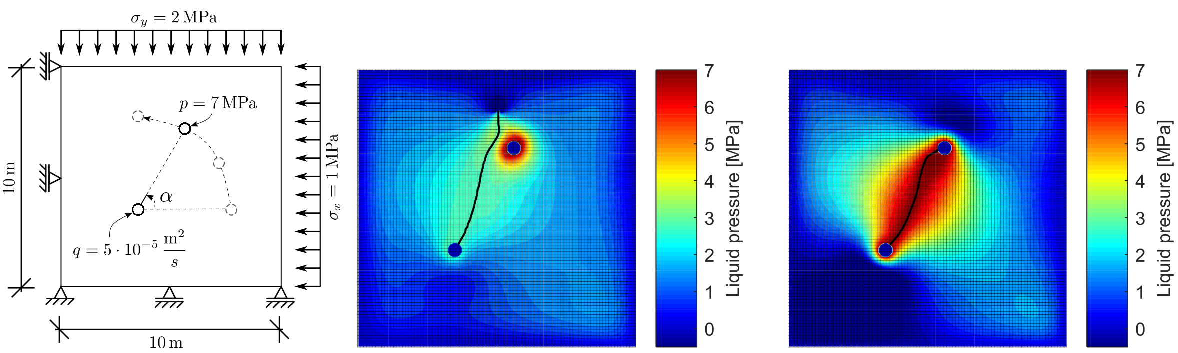


Figure 2. IGFEM - enrichment functions and element partitioning

3 Application

In the following example, a 2D analysis of hydraulic fracturing in a domain with two boreholes is analyzed by means of the presented method.

Figure 3 a) shows the geometry, loading, boundary conditions and material. The in situ stress state is represented by the external stresses σ_x and σ_y. The top borehole is loaded with a constant fluid pressure p and a constant fluid flux q is injected into the bottom left borehole, where a crack is initiated. The influence on the crack path is observed, while the position of the top right borehole is varied and the initial crack angle α is changed.



(a) Geometry and boundary conditions (b) $\alpha = 60°$ (c) $\alpha = 50°$

Figure 3. Hydraulic fracturing analysis of a 2D domain with two boreholes: Liquid pressure and crack path obtained for two different initial crack angles α. Material data: $E = 30\,\text{GPa}$; $\nu = 0.3$; $K_{IC} = 3\,\text{MPa}/\sqrt{s}$; $k_p = 10^{-16}\,\text{m}^2$; $\mu = 10^{-9}\,\text{MPa s}$. All edges are drained.

The liquid pressure and crack path for the case with an initial crack angle α = 60° is shown in Figure 3 b). As can be observed, the crack is deflected and influenced by the external loading and the pressurization of the top borehole. The crack aligns with the direction of the larger stress σ_y and passes by the top borehole. The pressurized top borehole attracts the crack, however, the pressure is not large enough that the crack penetrates into this borehole.
In hydraulic fracturing it is required to connect the boreholes. Consequently, a couple of modifications are possible to make the crack reach the top right borehole, e.g. changing the angle of the initial crack, the location of the borehole, such that the in situ stress state changes, or increasing the borehole pressure p. In Figure 3 c) the liquid pressure and crack path is shown for the case with a varied initial crack angle of α=50°. As the crack path takes a slightly longer way up to the top right borehole, the crack now is fully attracted by the borehole pressure, so that the boreholes are connected.

Conclusions

A computational method for hydraulic fracturing simulations based on an explicit fluid flow in the fracture and a strong coupling via Dirichlet boundary conditions has been proposed. The interface enriched generalized finite element method (IGFEM) is used to enrich the pore pressure field with a gradient jump and to prescribe pressure values inside the fracture or on arbitrary interfaces such as boreholes. In the presentation, several examples with multiple boreholes and external loadings are analyzed to demonstrate the viability of the proposed method.

Acknowledgements

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