A study of temperature and strain-rate dependent glass fracture behaviour

Ziyuan Li^{1*}, Yousef Heider¹ and Bernd Markert¹

Micro Abstract

In this contribution, the numerical simulation of brittle fracture of glass materials under room temperature is carried out on a continuum-mechanical scale using the theory of linear elasticity and J-integral theory, extended by a phase-field modelling (PFM) approach. Following this, behaviours such as crack nucleation, propagation, and bulk material fracture can be realised. Moreover, fracture stresses and J-integral values are compared to analyse material toughness.

¹Institute of General Mechanics, RWTH Aachen University, Aachen, Germany *Corresponding author: ziyuan@iam.rwth-aachen.de

Introduction

The fracture of glass materials in hot forming processes can directly lead to production inefficiency. Taking the precision glass moulding process (PGM) as an example, the working temperature is near the glass transition temperature (T_g) and the glass material represents either linear elastic or viscoelastic behaviour, which leads to the temperature and strain-rate dependency of glass fracture behaviour. The fracture of glass is initiated by micro cracks and propagates under tensile stresses. One of the most commonly used experimental methods for the analysis of glass fracture is the three-point bending test, in which a glass specimen with a rectangular or circular cross section is bent until fracture. To describe the behaviour of glass fracture on a macroscopic scale under both linear elastic and visoelastic model assumptions, the approach based on the J-integral theory [6] is applied. In the mean time, the brittle crack of the glass specimen at ambient temperature is described using a phase-field modelling (PFM) approach [2, 4, 5]. A combination of the two approaches is realised by the critical Griffith energy release rate (G_c) [1], which will be further explained.

1 Mathematical modelling

1.1 Phase-field modelling

The starting point of the PFM is the Griffith's energy-based criteria for brittle fracture [1], which initiated the concept of the critical fracture energy density G_c that represents the energy required to create a unit area of fracture surface. The global potential energy function Ψ of a cracked linear elastic, isotropic solid material can be defined as the sum of the elastic strain energy Ψ_{el} and the crack energy Ψ_{cr} integrated over the whole spatial domain. This integration can be achieved by inclusion of the phenomenological phase-field variable ϕ to distinguish between the cracked ($\phi = 0$) and the undamaged ($\phi = 1$) states of the material:

$$\Psi = \Psi_{\rm el} + \Psi_{\rm cr} = \frac{G}{4\epsilon} (1-\phi)^2 + G\epsilon |{\rm grad}\phi|^2 + [(1-\eta)(\phi)^2 + \eta] \Psi_{\rm el}^+ + \Psi_{\rm el}^-.$$
 (1)

Herein, ϵ is the internal length that is related to the width of the transition area between the cracked and the unbroken states and η is the residual stiffness. The fracture energy part is obtained from the non-local quadratic approximation; while the elastic energy part is divided

into positive and negative parts, which represent tensile and compressive modes, respectively. We assume a dynamic state for the present problem and neglect the body forces. By applying the Ginzburg-Landau evolution equation [2], the momentum balance equation and the phase-field evolution equation can be expressed as

div
$$\boldsymbol{\sigma} = \rho \dot{\mathbf{v}}$$
 and $\dot{\phi} = \frac{\partial \phi}{\partial t} = -M \frac{\partial \psi}{\partial \phi} = -M [2(1-\eta)\phi \psi_{elast}^{+} - \frac{G_c}{2\epsilon}(1-\phi) - 2\epsilon G_c \operatorname{div}(\operatorname{grad}\phi)],$ (2)

where $\boldsymbol{\sigma}$ is the stress tensor, $\rho(\mathbf{x}, t)$ is the mass density and $\mathbf{v}(\mathbf{x}, t)$ is the velocity. $M \geq 0$ represents a scalar-valued kinetic parameter related to the interface mobility (time dependency). G_c is the critical Griffith energy release rate, which represents the critical fracture energy.

1.2 J-integral theory

The further application of the J-integral approach extends the material model to the viscoelastic range. The J-integral was first given by Rice [6] as a path-independent line integral around the crack tip, which takes the form

$$J = \int_{L} (W dx_2 - \mathbf{t} \cdot \frac{\partial \mathbf{u}}{\partial x_1} ds), \tag{3}$$

where $W(x_1, x_2)$ is the strain energy density, x_1 and x_2 are the coordinate directions, $\mathbf{t} = \boldsymbol{\sigma} \mathbf{n}$ is the surface traction vector, $\boldsymbol{\sigma}$ is the stress tensor, \mathbf{n} is the normal to the curve L, which is an integral curve surrounding the crack tip, s is the crack length and \mathbf{u} is the displacement vector. For brittle fracture, a bridge is built by $J_c = G_c$ [1,6], where J-integral results are directly applied to PFM modelling. In 2D analysis, with the introduction of the finite element method and reasonable approximations, the numerical calculation of the J-integral can be further simplified to

$$J = \int_{A} \left[(\sigma_{xx} \frac{\partial u}{\partial x}) + \tau_{xy} \frac{(\partial v}{\partial x} - \omega) \frac{\partial q}{\partial x} + (\tau_{xy} \frac{\partial u}{\partial x} + \sigma_{yy} \frac{\partial v}{\partial x}) \frac{\partial q}{\partial y} \right] dA, \tag{4}$$

where q(x, y) must have a certain value on each node of the integral area, which takes the value q = 1 on the internal boundary and q = 0 on the external boundary for a plane problem. For elastic materials, ω is the elastic strain energy. For viscoelastic materials, it represents the equivalent elastic strain energy, which is the total area under the uni-axial stress-strain curve.

2 Results and discussion

To study both brittle and ductile fracture behaviour of glass material, a square-shaped, double symmetric problem, which is based on the three-point bending experiment illustrated in Fig. 1, is presented. The glass specimen is subjected by a constant strain rate from the presser downwards until a crack occurs. The glass specimens used for the experiments are of a rectangular cross section. They are made of three different types of glass materials for comparison, namely, the glass material H-K9L, L-BAL42 and SQ1. The selection of the three materials is based on their representative characteristics from different chemical compositions as well as different T_q .

According to Table 1, for each of the 12 temperatures, one higher strain rate and one lower strain rate are selected, altogether 24 sets of experiments are performed. For each set of experiment, 15-30 bending tests are performed considering numerical accuracy.

The selection of the higher strain rates aims at obtaining the brittle fracture state; while the selection of the lower strain rates is most critical, where a special state needs to be found when the majority of the glass specimens are just about to relax (unbroken), however in the end still break. In this way, a peak into the glass behaviour in the brittle-ductile fracture transition region is realised.

By applying PFM, the progress of the crack of brittle fracture is illustrated in Fig. 2. The crack propagates perpendicular to the maximum principal stress, which agrees with many

| L-BAL42 | | H-K9L | | SQ1 | |
|-----------------------------|----------------|-----------------------------|----------------|-----------------------------|----------------|
| $\lg \eta \; [dPa \cdot s]$ | $T[^{\circ}C]$ | $\lg \eta \; [dPa \cdot s]$ | $T[^{\circ}C]$ | $\lg \eta \; [dPa \cdot s]$ | $T[^{\circ}C]$ |
| - | 20 | - | 20 | - | 20 |
| 14.5 | 511 | | | | |
| 12.5 | 560 (Tg) | 13.1 | 1075 (Tg) | 12.4 | 506 (Tg) |
| 12 | 575 | | | | |
| 11 | 600 | 11 | 1250 | 11 | 532 |
| 10 | 630 | | | | |

Table 1. Representative viscosity values and according temperatures of three glass materials



Figure 3. J_c value of glass H-K9L.

ductile behavio

630

observations as given in the literature. The plot of J-integral values of one glass material at different temperatures and strain rates is depicted in Fig. 3. A jump of the J_c value is found near the glass transition temperature T_g region, which represents the crack resistance rise and indicates the sensibility of glass fracture in manufacturing processes near T_g .

dynamic fracture

model of glass H-K9L.



Figure 4. Fracture stresses σ_c of glass HK9L comparison.

Further comparison is made by fracture stresses obtained from experiments and PFM in Fig. 4. Similar tendency exists with higher σ_c at higher strain rate for both cases. Special attention needs to be drawn by relating to the experimental data, in the temperature zone between 575 °C and 600 °C. If we observe the curves backwards, when the temperature drops from 600 °C to 575 °C, in the σ_c curve from experiments, no great changes are noticed in the fracture stress; however, the J_c curve shows a tremendous decline. This points out the fact that the fracture

resistance of the glass material drops rapidly during the cooling process when the temperature is near the glass transition zone and makes this process step especially sensitive and has high chances of glass fracture, even when the maximum fracture stress appears to be stable. This conclusion shows the privilege of the J-integral approach from energy point of view in comparison to experimental approach and gives good base to future process optimization.

Conclusions

In conclusion, this study presents a robust and comprehensive scheme for the modelling of glass fracture, which can be implemented in usual finite element codes. The glass fracture behaviour is proved to be strongly temperature and strain-rate dependent. The J-integral has been proved to be an effective tool for fracture analysis of not only brittle materials but also materials with ductility. The proposed phase-field model results in a good qualitative agreement with J-integral results and experimental data, in which the increased strain rate plays a key role in the fracture stress gain. The combined approaches can serve as a base for a new direction in future studies and real applications in the field of glass fracture. An important conclusion from this study is the locating of the fracture-sensitive region into the cooling process of glass moulding procedures, which shows good value in industrial applications.

This work, however, remains to have improvement areas and extensions, such as the description of viscoelastic fracture and the extension towards 3D problems. An outlook of this work could be the testing and comparison of different phase-field parameters and evolution approaches to better control the crack propagation. A close and continuous monitoring of the changes in the crack edge profile along with different load in the experimental approach could also be beneficial to improve accuracy. The implementation of a ductile phase-field dynamic fracture model for glass material is our next intention.

Acknowledgements

We thank Gang Liu from Fraunhofer Institute for Production Technology IPT, who provided insight and expertise that greatly assisted the research.

References

- A. A. Griffith. The phenomena of rupture and flow in solids. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, Vol. 221:p. 163–198, 1921.
- [2] C. Kuhn and R. Müller. A phase field model for fracture. Engineering Fracture Mechanics, Vol. 77:p. 3625 – 3634, 2010.
- [3] Z. Li, G. Liu, and F. Klocke. Experimental and numerical studies of temperature and strain rate dependent fracture behaviours. Aachen, 2016. Insitute of Werkzeugmaschinenlabor WZL der RWTH Aachen University.
- [4] C. Miehe, M. Hofacker, and F. Welschinger. A phase field model for rate-independent crack propagation: robust algorithmic implementation based on operator splits. *Computer Methods* in Applied Mechanics and Engineering, Vol. 199:p. 2765–2778, 2010.
- [5] S. P. Patil, Y. Heider, C. A. H. Padilla, E. R. Cruz-Chú, and B. Markert. A comparative molecular dynamics-phase-field modeling approach to brittle fracture. *Computer Methods in Applied Mechanics and Engineering*, Vol. 312:p. 117–129, 2016.
- [6] J. R. Rice. A path independent integral and the approximate analysis of strain concentration by notches and cracks. *Journal of Applied Mechanics*, Vol. 35:p. 379–386, 1968.