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Reduced order modeling of the viscoelastic properties of asphalt concrete

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Micro Abstract
Mastic asphalt is a highly heterogeneous mixture consisting of elastic mineral aggregates and of a viscoelastic bituminous binding agent. To obtain the mixture's overall material behavior, we apply computational homogenization and order reduction. We computationally generate statistical volume elements based on real-data from X-Ray Computed Tomography. The resulting macroscopic material model is used to verify the method in comparison to laboratory experiments.

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Introduction

In this study, we computationally investigate the material properties of biphasic asphalt concrete which appears as a complex mixture containing mineral aggregates and a bituminous binding agent [5]. The mineral aggregates can be seen as linear-elastic particles that are surrounded by a visco-elastic bituminous suspension so that the geometry is highly heterogeneous. Because the overall material behavior depends on the microstructural geometry, it cannot be predicted by only knowing the material properties of the two components. As a consequence, we generate the microstructure computationally based on structural parameters extracted from X-ray computed tomography (XRCT) scans like volume fraction and particle size distribution of the mineral aggregates to obtain statistical volume elements (SVEs). These SVEs are assumed to be sufficiently representative for the overall viscoelastic material behavior of the asphalt concrete [5]. For the computational homogenization of the asphalt concrete, we apply the nonuniform transformation field analysis (NTFA) to obtain the overall material behavior of these SVEs [2]. The effective viscoelastic material model is validated with laboratory experiments.

Computational generation of the SVEs

This section provides information about the computational generation procedure to generate artificial microstructures of asphalt concrete. The generation process starts with the Lubachevsky-Stillinger algorithm [1] to determine the allocation of the mineral aggregates in the cubic SVE. This algorithm derives a dense sphere packing by calculating the motion of spheres with random initial positions and random initial velocities within a periodic cube. The spheres grow with a certain growth rate according to the final particle size distribution and collide fully elastically with each other. This algorithm stops when the time between the collisions falls below a certain limit so that we obtain the allocations and the radii of densely packed spheres [1]. Afterwards, we apply the weighted Voronoi tessalation as seen in [2] to generate particles that represent the mineral aggregates. Therefore, we take the allocation of the spheres as seed points for this calculation and the radii as the weighting factors to adjust the particle sizes. The points of the triangulated surface of the resulting particles are statistically shrunk towards their centers

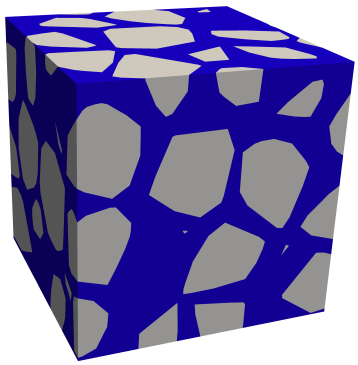

Figure 1. SVE containing mineral aggregates (grey) and bituminous binding agent (blue)

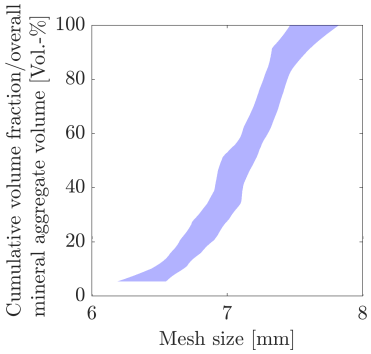

Figure 2. Standard deviation of particle size distributions of an ensemble consisting of 10 microstructures

of mass with a scale factor that varies between two chosen values to obtain the wanted mass fractions. To generate a periodic geometry, we arrange this set of particles of one unit cell 27 times to a $3 \times 3 \times 3$-sized cube. Afterwards we intersect this new arrangement with a cubic unit cell in the center of it to obtain the geometry of the mineral aggregates within the SVE [5]. The rest of the unit cell is assumed to contain the bituminous binding agent as seen in Fig. 1. Due to the statistical variance of the results from the Lubachevsky-Stillinger algorithm and the stochastic shrinking procedure, the resulting geometries and their characteristic particle size distributions and volume fractions vary as well, even if the same input parameters for the whole generation process have been chosen as seen in Fig. 2. To use these structures for the computational homogenization process, we mesh the SVEs periodically [5].

Material properties

This section describes the material properties of the two constituents of the investigated biphasic asphalt concrete. The concrete particles are assumed to be linear-elastic with a bulk modulus $K^s = 56$ GPa and a shear modulus $G^s = 32$ GPa as seen in Fig. 4, which are investigated material constants for Diabas rock [5]. The binding agent behaves visco-elastically and is described with the generalized Maxwell-Zener model which presumes a linear spring with a bulk modulus K^0 and a shear modulus G^0 and multiple Maxwell chains with the shear moduli G^i and relaxation times τ^i ordered in a parallel manner. We identify these material parameters in Fig. 3 by fitting the master curve [6] of the Maxwell model to the results from dynamic shear rheometer tests under the assumption that bitumen behaves almost fluid-like. Hence, we consider the equilibrium shear stiffness G^0 to be small compared to the non-equilibrium shear stiffness G^i with $i \neq 0$ of each Maxwell chain [5]. Additionally, the bituminous phase can be seen as nearly incompressible because of the high bulk modulus K^0 compared to the shear stiffness.

i	0	1	2	3	4
G^i [MPa]	0.0282	2.822	17.98	77.76	222.9
K^i [MPa]	17321	0	0	0	0
τ^i [s]	—	6.341	0.6341	0.06341	0.006341

Figure 3. Material data of the Maxwell-Zener model for bituminous phase

G^s [MPa]	32019
K^s [MPa]	56032

Figure 4. Material data of mineral aggregates

Nonuniform transformation field analysis

For the computational order reduction process, the nonuniform transformation field analysis [2–4] is applied. Therefore, several load cases of the SVEs under periodic boundary conditions (PBC) are simulated with the finite element method (FEM) as seen in Fig. 5. These training computations serve to generate snapshots of the observed viscous strains. We use these snapshots to derive a reduced basis of n viscous strain modes via Proper Orthogonal Decomposition (POD). According to the NTFA algorithm proposed in [3], we compute the linear-elastic response for zero viscous eigenstrains. Moreover, we solve n eigenstress problems controlled by the viscous strain modes at zero overall deformation. Thus, we are able to determine the microscopic constitutive model including the effective viscoelastic evolution equation of the biphasic mixture. All Finite Element computations are executed in ABAQUS/STANDARD.

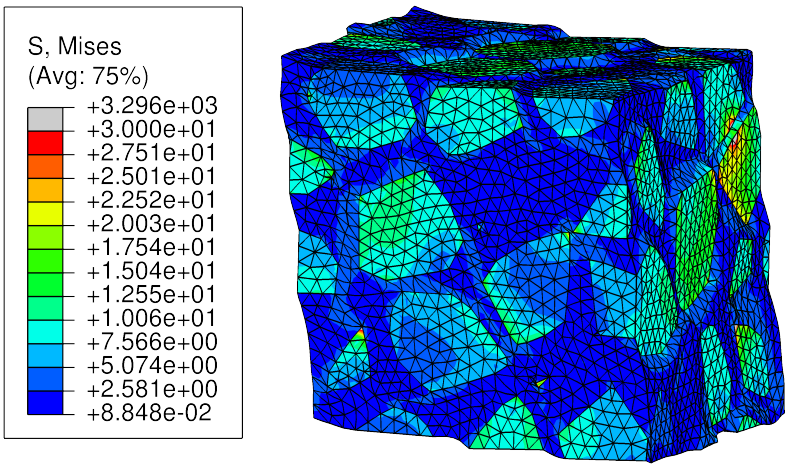

Figure 5. Deformed SVE with an average shear $\bar{\varepsilon}_{12} = 0.01$

Indirect tension tests

For the verification of the homogenized material properties, we execute indirect tension tests on the macroscale as seen in Fig. 6 and simulate them using the output of the discussed NTFA analysis. Therefore, cylindrical asphalt concrete specimens with known sieve curves of the mineral aggregates and known volume fractions are produced physically and computationally. The specimens undergo a displacement controlled inhomogenous stress relaxation test. The resulting forces of the loading surfaces as well as the lateral displacements are evaluated as shown in Fig. 6.

Conclusion

This study shows the computational generation and homogenization of asphalt concrete structures. Statistical volume elements based on geometric data from XRCT scans could be produced within a statistical variance. These structures were homogenized to obtain their overall material behaviors. Therefore, a NTFA-technique for viscoelastic composites is employed. The resulting macroscopic constituent relations of the mixture are used to simulate an indirect tension test. The simulation based on statistically generated volume elements and viscoelastic NTFA-technique are validated with corresponding laboratory experiments.

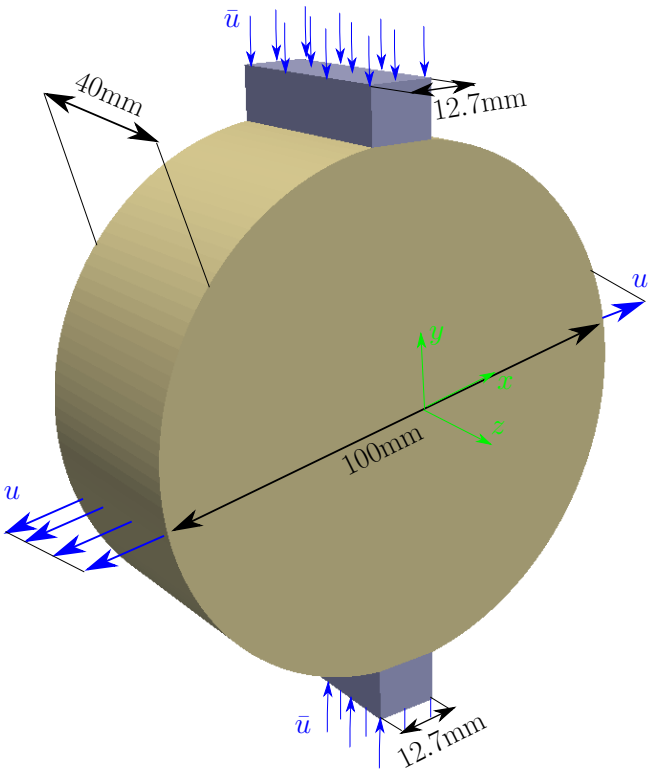

Figure 6. Indirect tension test with prescribed displacement $\bar{u}$

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