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Study of the Regularization Scheme of an Advanced Rock Model

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Micro Abstract
Quasi-brittle materials such as rock exhibit strain softening in the post-peak region leading to failure. The application of an advanced constitutive model for rock predicting irreversible deformation, strain hardening and strain softening is discussed. The aim is to the study the regularization scheme of the rock model based on the over-nonlocal implicit gradient enhancement in numerical simulations of a biaxial compression test.

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Introduction

During the construction of a deep tunnel, reliable estimates of displacements of the tunnel surface and of loads acting on the support structure are of major interest. The complex mechanical behavior of the rock-support system during tunnel advance can be analyzed by means of numerical simulation methods, e.g., the finite element method, where modeling the highly nonlinear mechanical behavior of rock plays a significant role. Rock is classified as a frictional cohesive material characterized by nonlinear stress-strain behavior including softening in the post-peak region leading to failure. In [10], a damage plasticity model for intact rock formulated in the framework of continuum mechanics was proposed. This rock model captures irreversible deformation, strain hardening, stiffness degradation and strain softening. It is common knowledge that strain softening is a structural phenomenon and may yield mesh-dependent results in finite element simulations without regularization of the softening behavior. In order to ensure objective results upon mesh refinement, in the aforementioned rock model the concept of the mesh-adjusted softening modulus from the crack-band theory of Bažant and Oh [1] was adopted. Nevertheless, the consistent determination of the characteristic element length especially for higher order or distorted elements remains questionable and localization zones are biased by the orientation of the finite element mesh, as discussed in [5]. Hence, in the present contribution, the regularization scheme of the aforementioned rock model is modified by means of the adoption of the over-nonlocal implicit gradient-enhancement, as presented in [8]. The capability of predicting shear failure in a mesh-insensitive manner is assessed in finite element simulations of a biaxial compression test.

1 Constitutive Model for Intact Rock and Rock Mass

An isotropic damage plasticity model describing the nonlinear mechanical behavior of intact rock was proposed in [10]. The rock model adopts the framework of plasticity theory in the effective stress space in conjunction with the theory of damage mechanics and is able to represent linear elastic and non-associated plastic material behavior, nonlinear isotropic strain hardening and nonlinear isotropic strain softening, the latter modeled by means of a scalar isotropic damage variable. The nominal stress $\boldsymbol{\sigma}$ is calculated from the effective stress $\bar{\boldsymbol{\sigma}}$ by means of $\boldsymbol{\sigma} = (1-\omega)\,\bar{\boldsymbol{\sigma}}$, where ω denotes the scalar isotropic damage variable ranging from $\omega = 0$

(undamaged material) to $\omega = 1$ (fully damaged material). The effective stress $\bar{\boldsymbol{\sigma}}$ is related to the elastic strain $\boldsymbol{\varepsilon}^{\mathrm{e}} = \boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{p}}$, determined from the total strain $\boldsymbol{\varepsilon}$ and the plastic strain $\boldsymbol{\varepsilon}^{\mathrm{p}}$, by the elastic stiffness tensor $\mathbb{C}$ leading to the stress-strain law in the total form

$$\boldsymbol{\sigma} = (1-\omega)\,\mathbb{C} : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^{\mathrm{p}})\,. \tag{1}$$

The evolution law of ω, defined as

$$\omega(\alpha_{\mathrm{d}}) = 1 - \exp\left(-\alpha_{\mathrm{d}}/\varepsilon_{\mathrm{f}}\right)\,, \tag{2}$$

depends on a strain-like internal softening variable α_{d}, whose evolution is linked to the plastic strain rate, and the softening modulus ε_{f}. To account for the nonlinear mechanical behavior in predominantly hydrostatic compression, the initial yield surface is characterized by a cap. Nonlinear isotropic hardening is described by an evolving yield surface in the effective stress space driven by a strain-like internal hardening variable. The attained yield surface in the ultimate hardening state corresponds to the failure criterion of Hoek and Brown [3] in the smooth version proposed by Menétrey and Willam [6]. Furthermore, the intact rock model can be extended to rock mass using empirical down-scaling factors of the rock mass classification system of Hoek and Brown [4].

2 Regularization by the Over-Nonlocal Gradient Enhancement of the Rock Model

To provide objective results in terms of mesh refinement, mesh orientation and interpolation order of finite elements, the original damage plasticity model for intact rock in [10] is enriched by a nonlocal field of the damage-driving variable $\bar{\alpha_{\mathrm{d}}}$, which is implicitly defined as the solution of a Helmholtz-like partial differential equation with the local field of the strain-like softening variable α_{d} as the source term. Hence, the governing equations in the strong form yield a fully coupled system of second-order partial differential equations

$$\boldsymbol{\nabla} \cdot \boldsymbol{\sigma} + \boldsymbol{f} = \boldsymbol{0}\,, \tag{3}$$

$$\bar{\alpha_{\mathrm{d}}} - l^2\,\boldsymbol{\nabla}^2 \bar{\alpha_{\mathrm{d}}} = \alpha_{\mathrm{d}}\,, \tag{4}$$

composed of the equilibrium equation and the Helmholtz-like partial differential equation, respectively. The material parameter l describes a length scale which controls the size of the fracture process zone and acts as a localization limiter. In addition, an averaged strain-like softening variable $\hat{\alpha_{\mathrm{d}}}$ is calculated from the weighted sum of the local and nonlocal softening variable by the weighting factor m and is expressed as

$$\hat{\alpha_{\mathrm{d}}} = m\,\bar{\alpha_{\mathrm{d}}} + (1-m)\alpha_{\mathrm{d}}\,. \tag{5}$$

Values for $m > 1$ correspond to the over-nonlocal formulation, which was originally introduced to provide full regularization, as has been shown in, e.g., [2]. Finally, the averaged strain-like softening variable replaces the local strain-like softening variable in (2) leading to the modified damage law written as

$$\omega(\hat{\alpha_{\mathrm{d}}}) = 1 - \exp(-\hat{\alpha_{\mathrm{d}}}/\varepsilon_{\mathrm{f}})\,. \tag{6}$$

The general mathematical derivation and the detailed implementation framework for the adopted over-nonlocal implicit gradient enhancement can be found in, e.g., [7, 8].

3 Finite Element Simulations of a Biaxial Compression Test

The over-nonlocal implicit gradient enhancement of the damage plasticity model for intact rock is verified in finite element simulations of a biaxial compression test under plane-strain conditions, which was analyzed in [10] on the basis of the mesh-adjusted softening modulus.

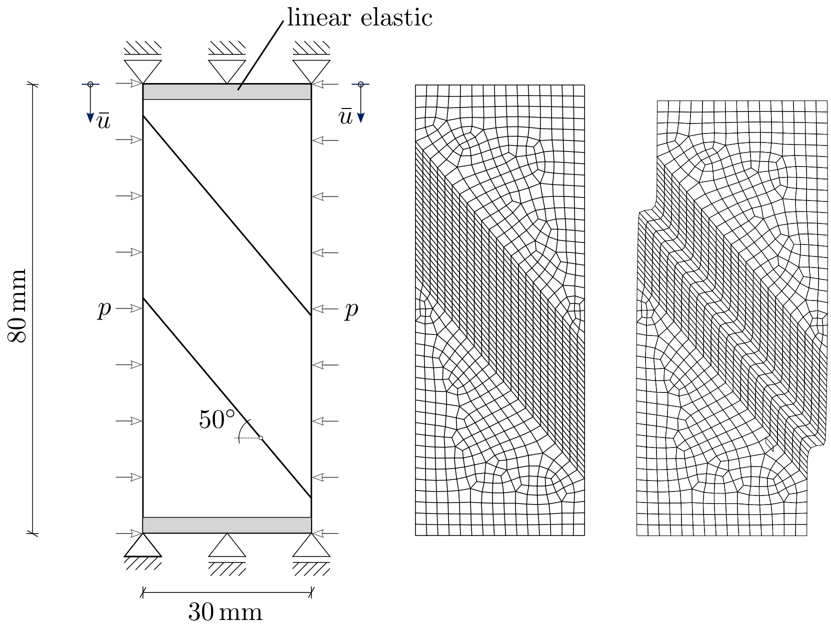

Figure 1. Model of the biaxial compression test (left), finite element discretization with element size of 1 mm in the vicinity of the localization zone in the undeformed (center) and the deformed configuration (right).

In this benchmark test for predicting the formation of shear bands, a rectangular specimen of non-burst-prone Donbass sandstone is considered. The employed material parameters of the rock model are taken from [10]. An associated plastic flow rule is considered to avoid localization due to non-associated plastic flow before entering the softening domain. The additional parameters required for the over-nonlocal implicit gradient-enhanced model, l, m and ε_{f}, are chosen as 2.5 mm, 1.05 and 0.0067 respectively. The model geometry and boundary conditions are depicted in Figure 1. The top and bottom row of finite elements are modeled by linear elastic material behavior. For the nonlocal damage-driving variable homogeneous Neumann boundary condition ($\boldsymbol{n} \cdot \boldsymbol{\nabla} \bar{\alpha}_{\mathrm{d}} = 0$) is assumed at the boundary surface, as proposed in [8].

As this test is simulated under plane strain conditions, the element edges in the vicinity of the localization zone are aligned with the expected shear band to avoid spurious locking, as reported in [9]. The specimen is discretized by fully integrated quadrilateral elements using linear shape functions. Three different sizes of finite elements in the localization zone are investigated: 1 mm, 2 mm and 4 mm. To trigger localization, the material parameters of a small zone of elements on the right boundary of the inclined mesh are slightly reduced.

In the first simulation step, hydrostatic initial stresses of $\sigma_{11}^{(0)} = \sigma_{22}^{(0)} = \sigma_{33}^{(0)} = -50\,\mathrm{MPa}$ are applied, which are in equilibrium with the lateral pressure $p = 50\,\mathrm{MPa}$. In the second simulation step, an incrementally increasing uniform vertical displacement at the top boundary of the specimen is prescribed up to $\bar{u} = 2\,\mathrm{mm}$. The deformed configuration of the specimen with finite element size of 1 mm is illustrated in Figure 1, right.

In Figure 2, the vertical reaction force at the top surface divided by the cross section of the specimen is shown for the different meshes in terms of the vertical displacement. Upon mesh refinement the obtained load-displacement curves converge towards a distinct result. For the coarse mesh, the element size of 4 mm is larger than the interaction radius l, which prohibits that the gradient of the nonlocal damage-driving variable can be represented sufficiently.

Conclusions

The damage plasticity model for intact rock and rock mass proposed in [10] was extended by the over-nonlocal implicit gradient formulation presented in [8] to ensure mesh-independent results.

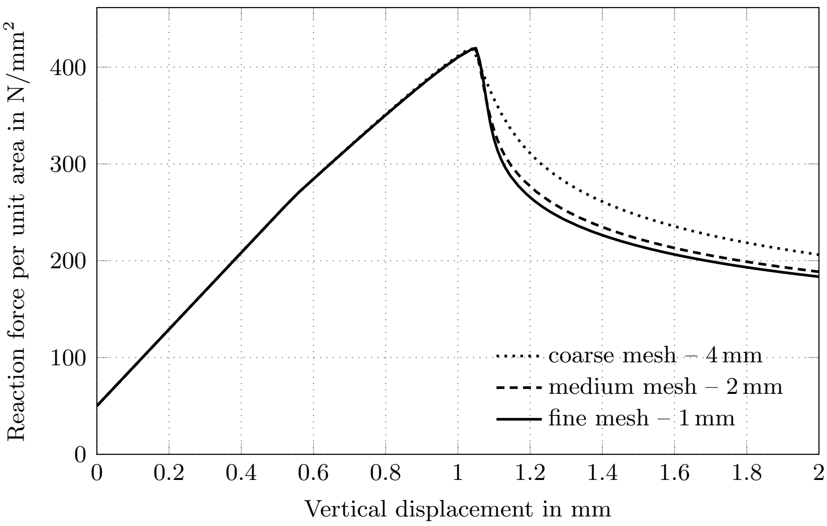

Figure 2. Comparison of the load-displacement curves of the biaxial compression test computed based on different sizes of finite elements.

The capability of the model to predict shear failure in an objective manner was shown based on finite element simulations of a biaxial compression test. As a next step the rock model enhanced by the over-nonlocal implicit gradient formulation will be employed in simulations of deep tunnel advance.

References

[1] Z. Bažant and B. Oh. Crack band theory for fracture of concrete. *Materials and Structures*, Vol. 16(3):p. 155–177, 1983.

[2] G. Di Luzio and Z. P. Bažant. Spectral analysis of localization in nonlocal and over-nonlocal materials with softening plasticity or damage. *International Journal of Solids and Structures*, Vol. 42(23):p. 6071–6100, 2005.

[3] E. Hoek and E. T. Brown. Empirical strength criterion for rock masses. *Journal of Geotechnical and Geoenvironmental Engineering*, Vol. 106(ASCE 15715):p. 1013–1035, 1980.

[4] E. Hoek and E. T. Brown. Practical estimates of rock mass strength. *International Journal of Rock Mechanics and Mining Sciences*, Vol. 34(8):p. 1165–1186, 1997.

[5] M. Jirásek and M. Bauer. Numerical aspects of the crack band approach. *Computers & Structures*, Vol. 110:p. 60–78, 2012.

[6] P. Menetrey and K. Willam. Triaxial failure criterion for concrete and its generalization. *ACI Structural Journal*, Vol. 92(3):p. 311–318, 1995.

[7] R. Peerlings, R. De Borst, W. Brekelmans, and J. De Vree. Gradient enhanced damage for quasi-brittle materials. *International Journal for Numerical Methods in Engineering*, Vol. 39:p. 3391–3403, 1996.

[8] L. H. Poh and S. Swaddiwudhipong. Over-nonlocal gradient enhanced plastic-damage model for concrete. *International Journal of Solids and Structures*, Vol. 46(25):p. 4369–4378, 2009.

[9] S. Rolshoven and M. Jirásek. Numerical aspects of nonlocal plasticity with strain softening. In *Computational Modelling of Concrete Structures-Proc. Europ. Conf. EURO-C*, pages 17–20, 2003.

[10] D. Unteregger, B. Fuchs, and G. Hofstetter. A damage plasticity model for different types of intact rock. *International Journal of Rock Mechanics and Mining Sciences*, Vol. 80:p. 402–411, 2015.