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Virtual tests based on model reduction strategies for fatigue analysis

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Micro Abstract
Virtual tests for fatigue considering a large number of cycles in perspective of continuum damage mechanics are generally avoided due to numerical expense. To tackle this problem, a Proper Generalised Decomposition model reduction technique in time and space, and a multi-time scale approach are proposed. These innovations used in a non-incremental LATIN framework, reduce the computational cost drastically and can be contemplated to perform virtual analysis of high-cycle fatigue tests.

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Introduction

Failure due to fatigue has a great importance for engineering applications [7]. As it is difficult to extrapolate the behaviour of a structure in fatigue from the experimental knowledge gained through uniaxial test or very low cycle fatigue behaviour, tests dedicated to fatigue analysis are required. But, they generally need high experimental effort and time, and so are considered expensive in comparison with main tests used to determine other mechanical properties [7]. For example, for a S-N curve established for a loading frequency of 30 Hz, the testing time is in the order of 60 full days [7]. To circumvent technical or financial barriers, virtual tests may be of large benefit. Classical approaches for predicting fatigue behaviour of materials are based on S-N curves, Palmgren-Miner linear damage rule, or Paris-Erdogan fatigue crack propagation theory. These approaches are efficient and well-established in engineering application. However, they are mainly empirical and may exhibit some inaccuracies for complex fatigue loads. To circumvent this problem, continuum damage mechanics could be used [6].

Continuum damage models are consistent with the established thermodynamic framework, such that the second law of thermodynamics is a priori satisfied. They are also flexible to consider complex fatigue and enhance an instantaneous description of the state of the material through dedicated internal variables. The non-linear problem can be solved by conventional numerical strategies [4]. However, solving such a problem is computationally expensive hence it is not possible to consider this model for a large number of cycles even for academic problems. Strategies based on cycle jumps or homogenisation for combined fatigue have been proposed in the literature [6]. In this contribution, the usage of an innovative model reduction strategy is discussed to perform virtual fatigue tests based on continuum damage models. The numerical challenge is due to the high-dimensional problem and the large number of load cycles.

1 Innovative numerical approach for continuous damage computation

The numerical strategy summarised here and employed for virtual tests is based on a non-incremental framework called the LATIN method combined with model reduction techniques.

First a separation of variables is used, and then a two-scale approach is enhanced for solving the time problem, which is the bottleneck for fatigue virtual tests.

1.1 LATIN method

The Large Time Increment method referred to as LATIN method exhibits the specific characteristic that the approximation of the quantity of interest is looked for at every iteration on a time-space domain [5]. It is not required to reach the convergence at time t_i before investigating the time t_{i+1}. Each iteration is thus composed of two steps as sketched on Fig. 1. The resolution of the evolution equations on a first hand and consequently the resolution of the mechanical equilibrium. Therefore, the numerical difficulties are separated, and dedicated numerical strategies can be used to optimise the resolution. This method, developed in the 1980s has been used for many non-linear problems such as plasticity, contact, or parametric studies. It provides a good convergence behaviour, but it is intrusive in the finite element code [5]. Here the extended version dedicated to damage computation as introduced in [1] is summarised.

Considering damage, the local stage which tackles non-linear local equations comprises the evolution equations as well as the elastic state law that is not linearisable due to damage. The global stage considers the linear equations, i.e. the admissibility conditions, the linear state laws and the non-linear state law for damage. The iterative solutions are searched with respect to the search directions B^+ and B^- for the local and global problem respectively.

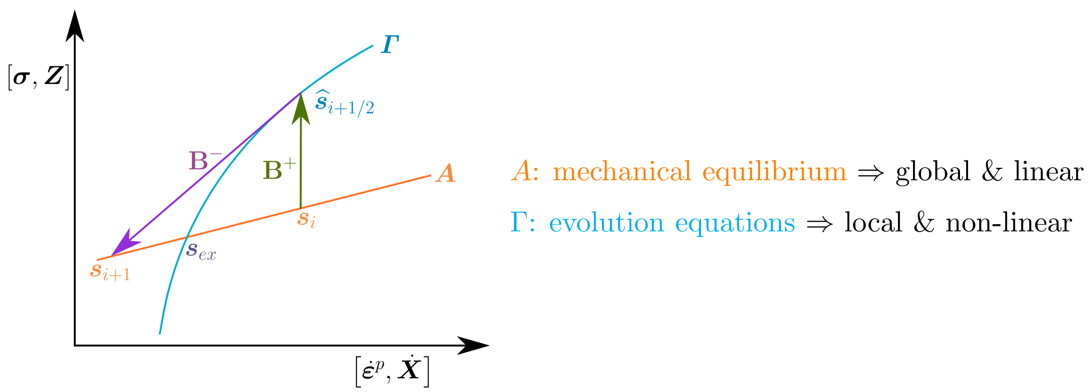

Figure 1. Two subsequent steps composing the LATIN iteration $i + 1$ (modified from [5])

Then, the major part of the numerical cost relies on the global problem. Therefore, the LATIN method is equipped with a model reduction technique introduced as the "radial approximation", also refered as the Proper Generalised Decomposition (PGD) [3]. For any quantity of interest, for example the plastic strain rate, the time and space dependencies are separated as represented on Fig. 2, and the approximation is written as a finite series such that

$$\dot{\varepsilon}^p(\underline{x}, t) = \sum_{j=1}^{\mu} \dot{\lambda}_j(t)\, \bar{\varepsilon}_j^p(\underline{x}), \tag{1}$$

where the number of PGD modes μ is automatically decided by the algorithm to satisfy the accuracy requirement. λ_j represents the time dependency whereas $\bar{\varepsilon}_j^p$ describes the spatial dependency. The computation does not require a training stage and the modes are computed on-the-fly.

Including damage in the LATIN-PGD framework, the stress is mathematically separated into a part depending on damage $\tilde{\sigma}$ and a part depending on plasticity σ' such that the plasticity-dependent part is also decomposed using PGD with the same time basis as the one used for the

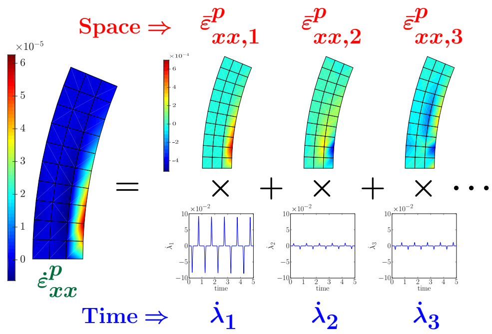

Figure 2. Decomposition of the plastic strain rate as a series of space and time functions

plastic deformation

$$\sigma'(x,y,t) = \sum_{j=1}^{\mu} \lambda_j(t)\,\bar{\sigma}_j(x,y) \tag{2}$$

and a dedicated spatial basis $\bar{\sigma}_j$. On an other hand, $\tilde{\sigma}$ can be obtained from the local stage.

Using LATIN-PGD method, the complexity of the problem is largely reduced. Contrary to most problems, for which remaining numerical challenges lie in the spatial problem, for fatigue analysis the time problem is too time-consuming to simulate more than a few thousands of cycles even for academic examples. Therefore, a finite-element like discretisation in time is proposed to solve the damage problem for thousands of cycles.

1.2 Two time scale discretisation

The time domain is discretised using two scales. Any time instant $t \in [0,T]$ can be written as

$$t = \theta_j + \tau_j, \tag{3}$$

where θ_j is the initial time of the cycle of interest and τ_j represents the time dependency within the cycle as illustrated in Fig.3.

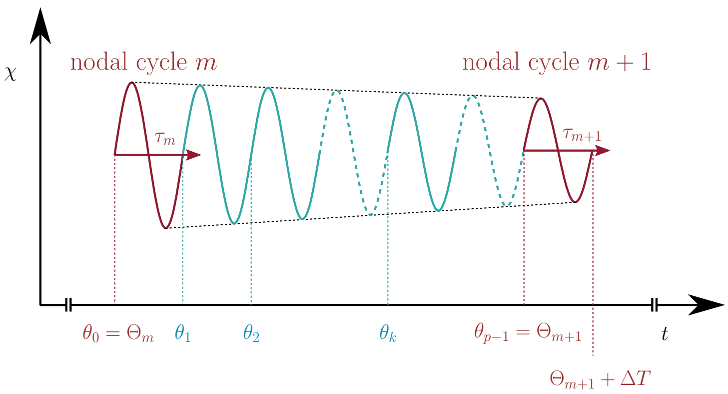

Figure 3. Finite-element like discretisation of the time domain based on two scales

The mechanical problem is solved only for some cycles called nodal cycles. Then, an interpolation between the temporal nodes as illustrated on Fig. 3 provides the estimation of the quantities of interest on the whole time domain with the fine time discretisation as with conventional finite element. For example, considering linear shape functions ν_m and ν_{m+1} defined respectively as,

$$\nu_m = \frac{\theta_j - \Theta_m}{\Theta_{m+1} - \Theta_m} \qquad \text{and} \qquad \nu_{m+1} = \frac{\Theta_{m+1} - \theta_j}{\Theta_{m+1} - \Theta_m}, \tag{4}$$

the interpolation within the element $[\Theta_m, \Theta_{m+1}]$ reads

$$\varepsilon_p(t) = \nu_m(\theta_j)\varepsilon_p(\tau_m) + \nu_{m+1}(\theta_j)\varepsilon_p(\tau_{m+1}). \tag{5}$$

A numerical difficulty is to estimate the initial value of the nodal cycles. For some discussion about this numerical aspect, it may be refered to [2].

2 Some virtual test results

The numerical strategy is applied to a fatigue test on a curved beam which is a quarter of an annulus, the geometry of which is described by the inner and outer radii ϕ_2 and ϕ_1 respectively. The beam whose geometry is depicted in Figure 4 is submitted to a cyclic load on the end section. The load is applied until reaching rupture or is limited to 10^5 cycles.

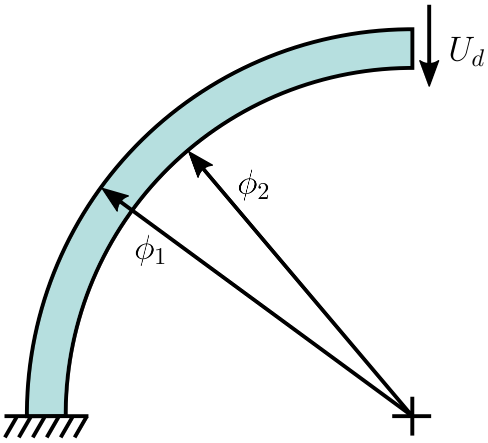

Figure 4. Geometry of the curved beam and definition of the load

Considering a sinusoidal load of the form $U_d = U_0 \sin\left(\frac{2\pi t}{T}\right)$, with U_0 being the amplitude, the damage distribution in the beam is depicted on Figure 5 for 100,000 cycles. The damage localises at the bottom of the beam. In the zone of interest, the damage evolution can be observed at the intermediate stages of 50,000 cycles, 75,000 cycles and the final stage of 100,000 cycles.

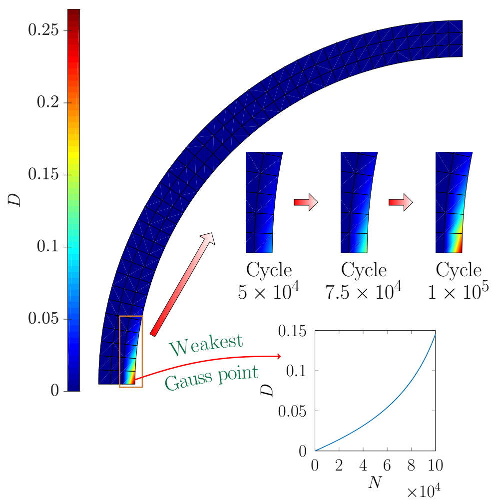

Figure 5. Damage evolution in a curved beam submitted to 100,000 load cycles

An evaluation of the computational cost and accuracy is given in Figure 6. Due to its computational cost, the mono-scale LATIN problem has been solved as a reference solution only up to

1000 cycles.

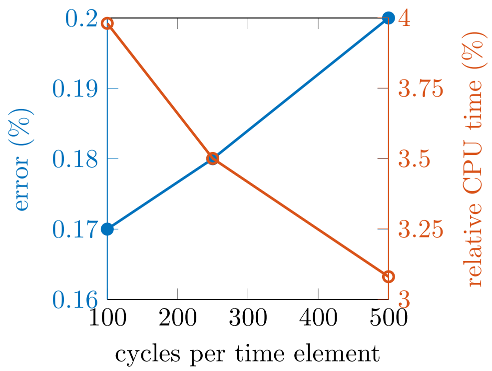

Figure 6. Computational effort and numerical accuracy for different temporal discretisation

Concerning the two time scale strategy with 200 cycles per element and for a total of 10,000 load cycles, a series of virtual tests is proposed to study the influence of the mean value of the loading (U_m) and initial damage (D_0) on the fatigue behaviour of the structure. The result of that campaign is depicted on Figure 7.

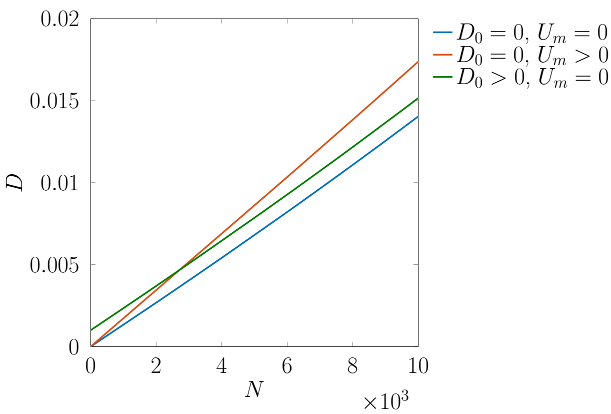

Figure 7. Damage evolution at the weakest Gauss point for different loading conditions

Conclusion

An advanced numerical strategy has been presented here to develop virtual fatigue tests. The numerical framework, which can be used for a large family of ductile damage behaviours, includes model reduction technique and a two time scale description which allow to simulate the test for a large number of cycles in a reasonable computational time. Thus a series of virtual tests can be planned for engineering parametric analysis. To investigate a larger domain of virtual tests including also high cycle fatigue problem, the numerical approach shall be equipped with a solver dedicated to brittle damage.

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