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Geometrically nonlinear single crystal viscoplasticity implemented into a hybrid discontinuous Galerkin framework

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Micro Abstract
The implementation of geometrically nonlinear crystal plasticity into a hybrid discontinuous Galerkin (DG) framework is presented using a regularization technique for very high strain rate sensitivity exponents. This combination leads to a numericallly efficient and locking-free model. The performance of the regularized hybrid DG crystal plasticity is examined on a planar single crystal.

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Introduction

There are many theories such as classical continuum (e.g., [10]) and strain-gradient theories (e.g., [1–3, 13, 15]) to study crystal deformations in large- and small-scale plasticity, respectively, which are in good agreement with the experimental data. For example, a comparison of a rate-dependent planar single crystal under tension and a rate-independent one using explicit numerical treatment shows that rate sensitivity delays the shear band development (see e.g., [6, 7]). Moreover, a hybrid discontinuous Galerkin (DG) method was introduced, for the first time, by Reed and Hill [8] to solve a linear first-order hyperbolic problem of neutron transport. Unlike conventional continuous methods, the DG framework allows displacement discontinuities between the element sub-domains. Later, a penalty term on the element boundaries was added to stabilize the solution [5].

1 Crystal viscoplasticity: dissipation and thermodynamically consistent flow rule

Assume the deformation mapping $\boldsymbol{x}(\boldsymbol{X}, t)$ is given, in which $\boldsymbol{X}$ and $\boldsymbol{x}$ are, respectively, position vector of a particle in the reference and current configuration at time t. The deformation gradient $\boldsymbol{F} = \partial\boldsymbol{x}/\partial\boldsymbol{X}$ is assumed to be decomposed multiplicatively, i.e., $\boldsymbol{F} = \boldsymbol{F}^{\mathrm{e}}\boldsymbol{F}^{\mathrm{p}}$ into elastic and plastic parts [4]. Regarding the continuum model of crystal viscoplasticity, the so-called plastic velocity gradient is given as a superposition of the contributions of the individual slip systems:

$$\boldsymbol{L}^{\mathrm{p}} = \sum_{\alpha=1}^{N} \dot{\gamma}_\alpha \boldsymbol{M}_\alpha, \tag{1}$$

in which N is the number of slip systems, $\dot{\gamma}_\alpha$ are the slip rates and $\boldsymbol{M}_\alpha = \boldsymbol{d}_\alpha \otimes \boldsymbol{n}_\alpha$ represent the crystal geometry in which $\boldsymbol{d}_\alpha$ and $\boldsymbol{n}_\alpha$ are the slip direction and slip plane normal vectors, respectively. In addition, the accumulated plastic strain reads:

$$\gamma_{\text{acc}} = \sum_{\alpha=1}^{N} \int_{0}^{t} |\dot{\gamma}_\alpha| \, \mathrm{d}t. \tag{2}$$

The dissipation per unit volume, represented in terms of the first Piola-Kirchhoff stress tensor $\boldsymbol{P}$ and the free energy ψ, is obtained by

$$\mathcal{D} = \boldsymbol{P} : \dot{\boldsymbol{F}} - \dot{\psi} \geq 0, \tag{3}$$

neglecting thermal effects.
Considering a suitable form of the free energy per unit volume (see e.g., [9]), one can assume a thermodynamically consistent flow rule as follows

$$\dot{\gamma}_\alpha = \operatorname{sgn}(\tau_\alpha)\dot{\gamma}_0 \left\langle \frac{|\tau_\alpha| - \tau^{\mathrm{c}}}{\tau^{\mathrm{D}}} \right\rangle^{p}, \tag{4}$$

in which $\dot{\gamma}_0$, τ^{c} , τ^{D} and p are the reference shear rate, the yield stress, the drag stress and the strain rate sensitivity, respectively. Moreover, τ_α is the resolved shear stress associated to slip plane α in the intermediate configuration.
The numerical solution of the nonlinear system of equations via the Newton scheme is challenging, especially for large values of the rate sensitivity parameter p. Therefore, a regularization technique is implemented by improving the starting guess in the Newton scheme (see [14] for details).

2 Hybrid discontinuous Galerkin framework

In contrast to the continuous finite element method, the displacement is not constrained to be continuous at the element boundaries in the hybrid DG framework (see Figure 1 and [11]). It is noteworthy to mention that the interface and the subdomains are not kinematically coupled. For further details concerning the method and its implementation see [11].

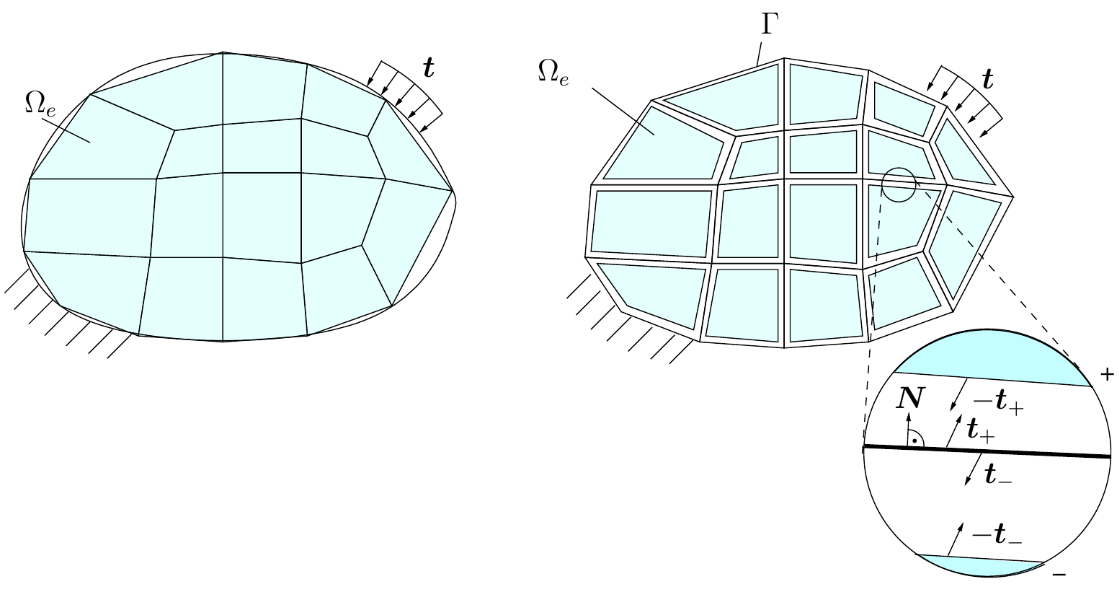

Figure 1. Left: Division of the body into subdomains. Right: Illustration with shrunk subdomains (Fig. taken from [11]).

3 Example

A specimen with dimensions 20×60 mm is considered being under uniaxial tension in longitudinal direction. The material is assumed to be elastically isotropic with Lamé parameters $\lambda = 35104.88$ MPa and $\mu = 23427.25$ MPa. The other material parameters are shown in Table 1 (see [12] for details). Double slip is investigated here where the slip directions are $\pm 30°$ about the y-axis. Figure 2 shows the distribution of the accumulated plastic strain at elongation 5.5 mm where $\theta = 10^{-1}|\partial\Omega_e|^{-1}$ MPa, in which $|\partial\Omega_e|$ is the sum of the element edge lengths.

τ_0 [MPa]	τ_∞ [MPa]	h_0 [MPa]	h_∞ [MPa]	$\dot{\gamma}_0$ [s^{-1}]	τ^{D} [MPa]	$p[-]$
0.84	49.51	541.48	1	10^{-3}	60	250

Table 1. The material parameters.

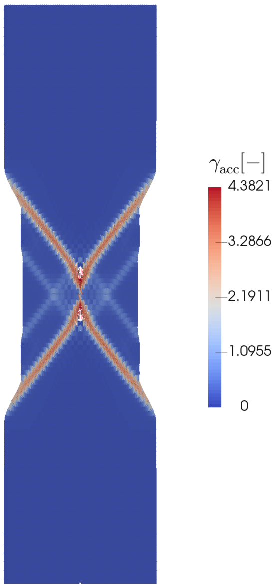

Figure 2. Distribution of the accumulated plastic (total number of elements is 80×120).

Conclusions

A geometrically nonlinear single crystal viscoplasticity model in combination with a DG formulation, here by quadrilateral hybrid DG elements with constant deformation gradient has been presented, leading to a numerically efficient model. In addition, a regularization method for the power law was applied to improve the numerical solution of the nonlinear equations.

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