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# Domain decomposition methods for fracture mechanics problems and its application to fiber reinforced concrete

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**Micro Abstract**
A finite element tearing and interconnecting (FETI) approach for phase-field models and gradient enhanced damage models is presented. These diffusive crack models can solve fracture mechanics problems by integrating a set of partial differential equations and thus avoid the explicit treatment of discontinuities. However, they require a fine discretization in the vicinity of the crack. FETI methods distribute the computational cost among multiple processors and thus speed up the computation.

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## Introduction

Strain hardening ultra high performance fiber reinforced cementitious composites (UHPFRCC) exhibit increased strength, ductility, and energy absorption capacity when compared to their quasibrittle, unreinforced counterparts [10]. A mesoscale finite element model can depict the underlying causes for the structural response of UHPFRCC and thus help to optimize the fiber content, the fiber dimensions, and the fiber orientation. The mesoscale model can either be used directly or as a representative volume element for a macroscopic model. We present a two-dimensional and a threedimensional mesoscale finite element model to simulate the structural response of strain hardening UHPFRCC. The mesoscale model employs an implicit gradient enhanced damage model [8] for the cement matrix and a local bond stress-slip model for the bond between the cement matrix and the steel fibers. The steel fibers are modeled discretely as one-dimensional truss elements that are coupled to the cement matrix via bond elements. The tensile stress-strain response of UHPFRCC is a consequence of local matrix cracking and bond failure. Both phenomena can be depicted when modeling the cement matrix, the steel fibers, and the fiber-to-matrix bond explicitly.

phase-field models [7] and gradient enhanced damage models are adopted to model fracture and to predict crack initiation, propagation, merging, and branching through the computational domain. They integrate a set of partial differential equations for the system and thus avoid the explicit treatment of discontinuities. The main attributes of these approaches are their simplicity and generality. However, they require a fine discretization in the region where the crack evolves. A finite element tearing and interconnecting (FETI) [5] approach for the diffusive crack models is presented to distribute the computational cost among multiple processors and thus speed up the overall computation.

Section 1 introduces two approaches to model fracture in concrete, the implicit gradient enhanced damage model in section 1.1 and the phase-field model for brittle fracture in section 1.2. Section 2 describes the proposed mesoscale model for UHPFRC and section 3 introduces the FETI method.

# 1 Modeling fracture mechanics problems with PDEs

## 1.1 Implicit gradient enhanced damage model

An implicit gradient enhanced damage model [8] is implemented to model the progressive degradation of material stiffness in the cement matrix. The problem can be described by a set of two differential equations, the static equilibrium in eq. (1) and an evolution equation in eq. (2)

$$\nabla \cdot \boldsymbol{\sigma} = \mathbf{0} \qquad \boldsymbol{x} \in \Omega \tag{1}$$

$$e - l\nabla^2 e = \varepsilon_{\text{eq}} \qquad \boldsymbol{x} \in \Omega \tag{2}$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $e$ is the nonlocal equivalent strain, $l$ is a length-scale parameter, and $\varepsilon_{\text{eq}}$ is the local equivalent strain. In this isotropic damage model, an exponential damage function $d$ describes the propagation and coalescence of microcracks in the material. The stress-strain relation reads

$$\boldsymbol{\sigma} = (1-d)\boldsymbol{C}\boldsymbol{\varepsilon} \tag{3}$$

where $\boldsymbol{\varepsilon}$ is the infinitesimal strain tensor and $\boldsymbol{C}$ is the fourth order material stiffness tensor. The damage function $d$ depends on a history variable $\kappa$ that characterizes the maximum strain in the loading history of the material, i.e.

$$\kappa(t) = \max_{\tau \leq t} \; \varepsilon_{\text{eq}}(\tau) \tag{4}$$

A damage function that is commonly used in conjunction with this model is

$$d(\kappa) = 1 - \frac{\kappa_0}{\kappa}\left[1 - \alpha + \alpha \exp\left(-\beta(\kappa - \kappa_0)\right)\right] \tag{5}$$

where $\kappa_0$ is the threshold for damage initialization, $\alpha$ influences the residual damage, and $\beta$ the initial slope of the exponential function.
The local equivalent strain $\varepsilon_{\text{eq}}$ is the right hand side of eq. (2) and couples the equilibrium equations with the inhomogeneous Helmholtz equation. The choice of the equivalent strain definition influences the result significantly. The modified von Mises definition [2] is often applied for concrete.

$$\varepsilon_{\text{eq}} = \frac{k-1}{2k(1-2\nu)} I_1 + \frac{1}{2k}\sqrt{\left(\frac{k-1}{1-2\nu}\right)^2 I_1^2 + \frac{12k}{(1+\nu)^2} J_2} \tag{6}$$

In eq. (6), $I_1$ is the first invariant of the strain tensor, $J_2$ is the second invariant of the deviatoric strain tensor, $k$ is the ratio of compressive strength to tensile strength of the cement matrix, and $\nu$ is the Poisson's ratio of the cement matrix.

## 1.2 Phase-field models for brittle fracture

In many phase-field approaches to model brittle fracture, the process of crack initiation, propagation, and branching is governed by a minimization problem of the energy functional [1]

$$E = \int_{\Omega} \psi d\Omega + G_f \int_{\Gamma} d\Gamma \tag{7}$$

where $\psi$ is the elastic energy density function and $G_f$ is the material fracture toughness. Equation (7) can be regularized by introducing a crack phase-field parameter $d \in [0, 1]$. The crack phase-field parameter indicates the degradation of the material stiffness, hence we use the same

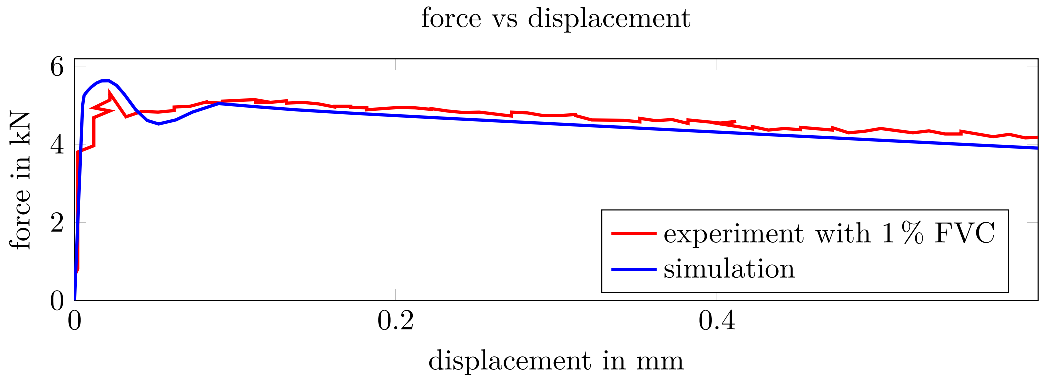



**Figure 1.** Experimental and numerical results of a uniaxial tension test on a dogbone specimen.

notation as for the damage function in the implicit gradient enhanced damage model, although they are two distinct quantities. The regularized energy functional is

$$E(\boldsymbol{u}, d) = \int_\Omega \psi(\boldsymbol{\varepsilon}(\boldsymbol{u}), d) + \frac{G_f}{2l}\left(d^2 + l^2 \nabla d \cdot \nabla d\right) d\Omega \tag{8}$$

where $l$ is a length-scale parameter. The variation of eq. (8) yields the strong form of the minimization problem

$$\nabla \cdot \frac{\partial \psi}{\partial \boldsymbol{\varepsilon}} = \boldsymbol{0} \qquad \boldsymbol{x} \in \Omega \tag{9}$$

$$\frac{G_f}{l}\left(d - l^2 \nabla^2 d\right) = \frac{\partial \psi}{\partial d} \qquad \boldsymbol{x} \in \Omega \tag{10}$$

In [7] the authors proposed the introduction of a history parameter $\kappa$ and an additive split of the elastic energy density $\psi = (1-d)^2\psi_+ + \psi_-$ for the right hand side of eq. (10) to ensure irreversibility and to inhibit crack initiation and propagation under compression.

$$\frac{\partial \psi}{\partial d} \Rightarrow 2(1-d)\kappa \tag{11}$$

$$\kappa(t) = \max_{\tau \leq t} \ \psi_+(\tau) \tag{12}$$

## 2 A mesoscale model for UHPFRC

The bond between steel fibers and cement matrix is significant for the postcracking behaviour of UHPFRC. A bond element is implemented to model the interaction between steel fibers and cement matrix. The cement matrix is meshed independently from the discrete steel fibers as proposed in [9]. The fibers are connected to the cement matrix via bond elements whose nodes are constrained to an element of the background mesh as shown in fig. 2.

The tensile strength of UHPFRC is influenced by the tensile strength of the steel fibers and the strength of the fiber-matrix bond. For short steel fibers, the latter is more important. The bond element proposed by [6] uses one-dimensional shape functions to calculate the relative displacement (slip) $\boldsymbol{s}$ between cement matrix and steel fibers. The Bertero-Popov-Eligehausen-model proposed by [3] defines the state of the bond as a function of the slip and can depict bond softening and fiber pullout. Figure 1 shows the force-displacement curve for a dogbone-specimen with 1 % fiber volume content under uniaxial loading conditions. After the linear elastic response, the concrete matrix starts to fail. Simultaneously, the steel fibers that bridge the damaged zone in the matrix are loaded until the fiber-matrix bond weakens and the fibers get pulled out of the matrix.

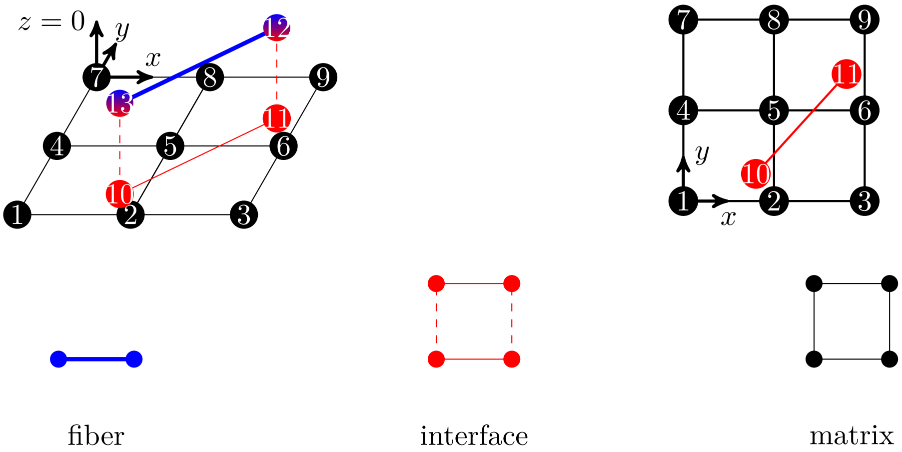
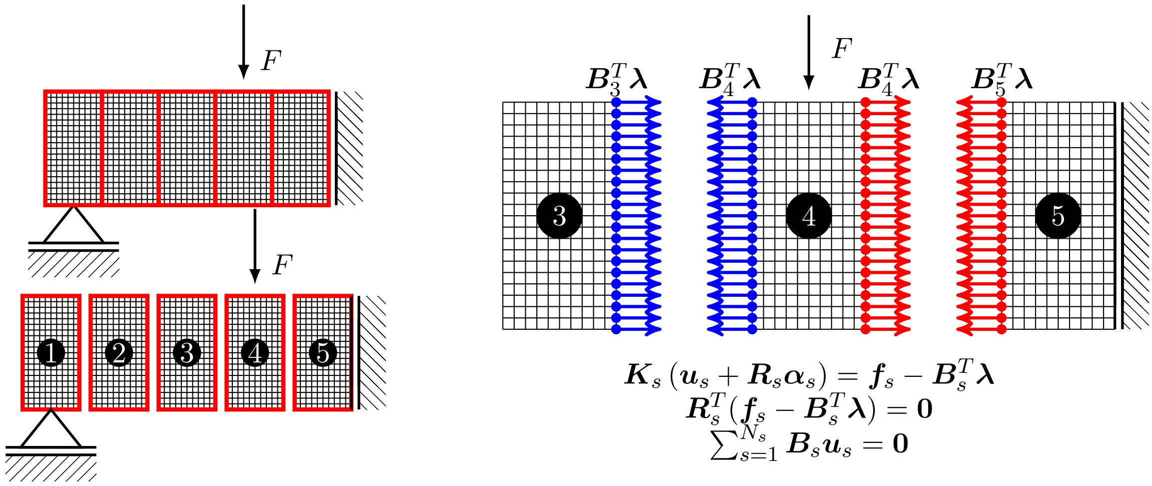



**Figure 2.** Bond between steel fibers and concrete matrix via an interface. The discretization of the concrete matrix is independent of the position and orientation of the fibers.




**(a)** Domain decomposition of a four point bending test into 5 subdomains. The local problem is fully parallelizable.

**(b)** Coupling of subdomains 3,4, and 5 using Lagrange multipliers $\boldsymbol{\lambda}$ and by enforcing the displacements at the subdomain interface to be zero.

**Figure 3.** Finite element tearing and interconnecting method for a four point bending test.

## 3 Finite element tearing and interconnecting method

The original finite element tearing and interconnecting (FETI) method [4] and its extensions are numerically scalable domain decomposition methods. All FETI methods use Lagrange multipliers to connect the subdomains at their corresponding interfaces. As shown in fig. 3, the idea is to split the domain into a number of subdomains and solve the local problem for each subdomain in parallel. To satisfy the continuity of the displacement, Lagrange multipliers are introduced that enforce the connection between neighboring subdomains. This method can significantly speed up the simulation. Figure 4 shows how the FETI method can be used to simulate a single edge notched tension test using the phase-field model from section 1.2.

## Conclusions

A mesoscale model for UHPFRC can depict the underlying causes for the structural response. However, a very fine discretization is necessary in the vicinity of the crack path and thus a high computational effort and memory requirement. Domain decomposition methods are a promising solution for this problem.

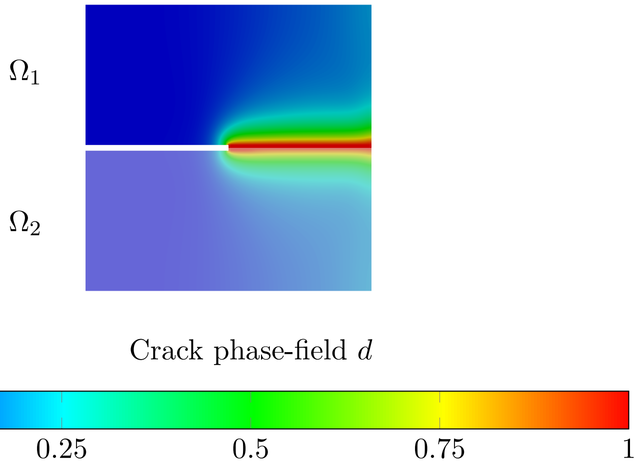



**Figure 4.** Crack phase-field at and $u_y = 7.5\,\mu\mathrm{m}$. Subdomain $\Omega_1$ opacity $= 1.0$. Subdomain $\Omega_2$ opacity $= 0.5$.

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